Глава 2. Расчетные задания.

Задача 1. Решить систему уравнений с двумя комплексными неизвестными z и w.

1.1. $\begin{cases} (1+i)z - 2w = -5 - i, \\ z - iw = 1 - i. \end{cases}$

1.2. $\begin{cases} 4z + iw = 5 + i, \\ iz + 2iw = 2 + 3i. \end{cases}$

1.3. $\begin{cases} 2z - iw = 3 + 2i, \\ iz + 2w = 1 + 3i. \end{cases}$

1.4. $\begin{cases} 4iz + w = -2 + i, \\ 2iz + 3iw = -5 + 6i. \end{cases}$

1.5. $\begin{cases} 2iz - w = -1 + 3i, \\ (1-i)z - 2w = -1 + i. \end{cases}$

1.6. $\begin{cases} (1-i)z - 2iw = 4, \\ 3iz + (2+i)w = -4 + 5i. \end{cases}$

1.7. $\begin{cases} 2z + 4iw = -4 + 2i, \\ z + (1+i)w = -1 + 2i. \end{cases}$

1.8. $\begin{cases} -3z + iw = -i - 1, \\ z - 4w = -8 - 3i. \end{cases}$

1.9. $\begin{cases} 4iz - w = -1 + 5i, \\ iz - 2w = -2 + 3i. \end{cases}$

1.10. $\begin{cases} 2iz + w = -2 + 3i, \\ -4iz + (1+i)w = 3 + 3i. \end{cases}$

1.11. $\begin{cases} z + (1+i)w = 3, \\ iz + 2iw = 2 + 3i. \end{cases}$

1.21. $\begin{cases} (i-1)z - 2iw = 1 - 5i, \\ 4iz + w = -2 + i. \end{cases}$

1.22. $\begin{cases} (1-i)z - 2w = -1 + i, \\ 4z + iw = 5 + i. \end{cases}$

1.23. $\begin{cases} -3z - (1-2i)w = -5 - 4i, \\ iz + 2w = -1 + 3i. \end{cases}$

1.24. $\begin{cases} 3iz - w = -3 - i, \\ z + (1+i)w = -1 + 2i. \end{cases}$

1.25. $\begin{cases} (1+i)z - 2w = -5 - i, \\ 4iz + w = -2 + i. \end{cases}$

1.26. $\begin{cases} 3z + 2w = 5 - 2i, \\ (1-i)z - 2w = -1 + i. \end{cases}$

1.27. $\begin{cases} 3z + 4iw = -1 + i, \\ (1+i)z + 2w = 4i. \end{cases}$

1.28. $\begin{cases} (1+i)z - w = -1, \\ -iz + 5w = 1 + 5i. \end{cases}$

1.29. $\begin{cases} 2z + iw = 3 + i, \\ iz + 2iw = 2 + 3i. \end{cases}$

1.30. $\begin{cases} 4iz - (1+i)w = -3 - 3i, \\ (1-i)z - 2iw = 4. \end{cases}$

1.31. $\begin{cases} 3iz + 2iw = 2 + 5i, \\ z + 2w = 3 - 2i. \end{cases}$

1.12. $\begin{cases} (1-i)z+iw=i, \\ 3z+iw=-1+3i. \end{cases}$

1.13. $\begin{cases} -iz+2iw=-3-i, \\ 3iz-4w=-1-i. \end{cases}$

1.14. $\begin{cases} 2z+4iw=-4+2i, \\ -z-5iw=5-i. \end{cases}$

1.15. $\begin{cases} iz+(i-1)w=-2-i, \\ (i-1)z-2iw=1-i. \end{cases}$

1.16. $\begin{cases} 4z-iw=2i+1, \\ 3iz+w=-1+i. \end{cases}$

1.17. $\begin{cases} 2z+3w=6+5i, \\ -iz+4iw=-3+8i. \end{cases}$

1.18. $\begin{cases} z+2iw=3+2i, \\ iz-(1-i)w=3i. \end{cases}$

1.19. $\begin{cases} 4iz-(1+i)w=-3-3i, \\ 3z+4iw=-1+i. \end{cases}$

1.20. $\begin{cases} iz-5w=-1-5i, \\ (1+i)z-2w=-1-i. \end{cases}$

1.32. $\begin{cases} iz-2w=-5+i, \\ 4z-iw=6-7i. \end{cases}$

1.33. $\begin{cases} 3iz-2w=3+i, \\ 6z-2iw=8+6i. \end{cases}$

1.34. $\begin{cases} 3iz+2iw=-5+2i, \\ -3z+2w=2-i. \end{cases}$

1.35. $\begin{cases} 2iz+4w=-2+4i, \\ iz+2iw=-3. \end{cases}$

1.36. $\begin{cases} 6iz-3w=6-3i, \\ 2z+6iw=-6-2i. \end{cases}$

1.37. $\begin{cases} (1+i)z-2iw=2, \\ iz+2w=3i+1. \end{cases}$

1.38. $\begin{cases} 4z-6iw=4i+10, \\ 5z+2iw=5i+3. \end{cases}$

1.39. $\begin{cases} (6+i)z+5w=i-1, \\ 3z+2iw=2+3i. \end{cases}$

1.40. $\begin{cases} 3iz+2w=5i+1, \\ 2z+3iw=8i-1. \end{cases}$

Задача 2. Найти модуль и главное значение аргумента комплексного числа.

2.1. $\dfrac{(\sqrt{3}-i)^6}{(1+i)^8}i^{174}$.

2.2. $\dfrac{(-\sqrt{3}+i)^7}{(-1+i)^6}i^{-33}$.

2.3. $\dfrac{(\sqrt{3}i+1)^6}{(1+i)^7}i^{168}$.

2.21. $\dfrac{(\sqrt{3}+i)^6}{(1-i)^8}i^{115}$.

2.22. $\dfrac{(1-i)^8}{(\sqrt{3}+i)^6}(-i)^{25}$.

2.23. $\dfrac{(\sqrt{3}-i)^6}{(1+i)^7}i^{198}$.

2.4. $\dfrac{(1-i)^9}{(\sqrt{3}-i)^6}(-i)^{51}$.

2.5. $\dfrac{(\sqrt{3}i+1)^7}{(1-i)^6}i^{-160}$.

2.6. $\dfrac{(1-i)^8}{(\sqrt{3}-i)^6}(-i)^{59}$.

2.7. $\dfrac{(1+i)^{10}}{(1-i)^8}i^{77}$.

2.8. $\dfrac{(\sqrt{3}+i)^6}{(1+i)^8}i^{201}$.

2.9. $\dfrac{(1+i)^7}{(-\sqrt{3}-i)^6}(-i)^{198}$.

2.10. $\dfrac{(\sqrt{3}i-1)^7}{(1-i)^6}i^{-67}$.

2.11. $\dfrac{(-\sqrt{3}-i)^7}{(-1+i)^8}i^{-90}$.

2.12. $\dfrac{(1-i)^6}{(-1+\sqrt{3}i)^7}i^{58}$.

2.13. $\dfrac{(1-i)^8}{(1+i)^{10}}i^{-29}$.

2.14. $\dfrac{(\sqrt{3}-i)^6}{(1-i)^8}i^{196}$.

2.15. $\dfrac{(1+i)^8}{(\sqrt{3}-i)^6}(-i)^{124}$.

2.24. $\dfrac{(\sqrt{3}i+1)^6}{(1-i)^7}i^{90}$.

2.25. $\dfrac{(\sqrt{3}i-1)^7}{(1+i)^6}i^{-18}$.

2.26. $\dfrac{(1+i)^6}{(-1+\sqrt{3}i)^7}i^{-34}$.

2.27. $\dfrac{(\sqrt{3}+i)^7}{(-1-i)^6}i^{21}$.

2.28. $\dfrac{(\sqrt{3}i-1)^6}{(1-i)^7}i^{109}$.

2.29. $\dfrac{(-\sqrt{3}+i)^7}{(1+i)^6}i^{-86}$.

2.30. $\dfrac{(1+i)^9}{(\sqrt{3}+i)^6}(-i)^{170}$.

2.31. $\dfrac{(1-i)^9}{(\sqrt{3}+i)^6}(-i)^{49}$.

2.32. $\dfrac{(\sqrt{3}+i)^8}{(1-i)^8}(-i)^{-85}$.

2.33. $\dfrac{(1+i)^9}{(1-i)^8}i^{-35}$.

2.34. $\dfrac{(\sqrt{3}-i)^6}{(1+i)^7}i^{-33}$.

2.35. $\dfrac{(1-i)^6}{(\sqrt{3}+i)^9}i^{74}$.

2.16. $\dfrac{(\sqrt{3}i+1)^7}{(1+i)^6}i^{-15}$.

2.17. $\dfrac{(1+i)^9}{(\sqrt{3}-i)^8}(-i)^{188}$.

2.18. $\dfrac{(1+i)^6}{(1+\sqrt{3}i)^7}i^{120}$.

2.19. $\dfrac{(1+i)^{10}}{(1-i)^7}i^{105}$.

2.20. $\dfrac{(\sqrt{3}i-1)^7}{(1+i)^6}i^{-18}$.

2.36. $\dfrac{(\sqrt{3}+i)^6}{(1-i)^4}i^{25}$.

2.37. $\dfrac{(\sqrt{3}i+1)^6}{(1-i)^8}i^{-14}$.

2.38. $\dfrac{(\sqrt{3}-i)^6}{(1-i)^7}i^{-49}$.

2.39. $\dfrac{(\sqrt{3}i-1)^6}{(1+i)^7}i^{-34}$.

2.40. $\dfrac{(1-i)^9}{(\sqrt{3}i-1)^6}(-i)^{-56}$.

Задача 3. Решить уравнение.

3.1. $z^4+1=0$.

3.2. $2z^4+1-i\sqrt{3}=0$.

3.3. $z^3-1=0$.

3.4. $z^3-i=0$.

3.5. $z^4-1=0$.

3.6. $2z^4+1+i\sqrt{3}=0$.

3.7. $z^3+1=0$.

3.8. $z^3+i=0$.

3.9. $z^4+16=0$.

3.10. $32z^4-1-i\sqrt{3}=0$.

3.11. $z^3-8=0$.

3.12. $z^3-8i=0$.

3.21. $16z^4-1=0$.

3.22. $z^4+8+i\cdot 8\sqrt{3}=0$.

3.23. $8z^3+1=0$.

3.24. $8z^3+i=0$.

3.25. $z^4+128-i\cdot 128\sqrt{3}=0$.

3.26. $z^3-27=0$.

3.27. $256z^4-1=0$.

3.28. $z^4+128+i\cdot 128\sqrt{3}=0$.

3.29. $27z^3-i=0$.

3.30. $z^4-256=0$.

3.31. $27z^3+i=0$.

3.32. $2z^4-1+i\sqrt{3}=0$.

3.13. $z^4 - 16 = 0$.

3.14. $32z^4 + 1 + i\sqrt{3} = 0$.

3.15. $z^3 + 8 = 0$.

3.16. $z^3 + 8i = 0$.

3.17. $16z^4 + 1 = 0$.

3.18. $z^4 + 8 - i \cdot 8\sqrt{3} = 0$.

3.19. $8z^3 - 1 = 0$.

3.20. $8z^3 - i = 0$.

3.33. $z^3 - 27i = 0$.

3.34. $256z^4 + 1 = 0$.

3.35. $z^4 - 8 - i \cdot 8\sqrt{3} = 0$.

3.36. $z^3 + 27 = 0$.

3.37. $z^4 + 256 = 0$.

3.38. $z^4 - 128 + i \cdot 128\sqrt{3} = 0$.

3.39. $z^3 + 27i = 0$.

3.40. $32z^4 - 1 + i\sqrt{3} = 0$.

Задача 4. Решить уравнение.

4.1. $e^z + 1 + i = 0$.

4.2. $\sin z - 4 = 0$.

4.3. $\mathrm{ch}z + 2 = 0$.

4.4. $3\,\mathrm{tg}\, z + 2\sqrt{3} - 3i = 0$.

4.5. $5\,\mathrm{cth}\, z - 3 + 4i = 0$.

4.6. $5\,\mathrm{ctg}\, z - 4 - 3i = 0$.

4.7. $3\,\mathrm{th}\, z - 3 - i \cdot 2\sqrt{3} = 0$.

4.8. $\cos z - 4i = 0$.

4.9. $2e^z - \sqrt{2} + i\sqrt{2} = 0$.

4.10. $\sin z + 3i = 0$.

4.11. $e^z - \sqrt{3} - i = 0$.

4.12. $\mathrm{ch}z - 3i = 0$.

4.13. $5\,\mathrm{tg}\, z - 3 - 4i = 0$.

4.14. $7\,\mathrm{cth}\, z - 8 - i \cdot 3\sqrt{3} = 0$.

4.15. $7\,\mathrm{tg}\, z - 3\sqrt{3} - 8i = 0$.

4.21. $e^z - 1 - \sqrt{3}i = 0$.

4.22. $\mathrm{th}\, z + 1 - 2i = 0$.

4.23. $\sin z + 5i = 0$.

4.24. $7\,\mathrm{ctg}\, z - 2\sqrt{3} - 3i = 0$.

4.25. $7\,\mathrm{th}\, z - 3 - i \cdot 2\sqrt{3} = 0$.

4.26. $5\mathrm{cth}z - 4 - 3i = 0$.

4.27. $e^z + \sqrt{2} + \sqrt{2}i = 0$.

4.28. $7\,\mathrm{tg}\, z - 3\sqrt{3} - 8i = 0$.

4.29. $\cos z + 3i = 0$.

4.30. $\mathrm{sh}z - 6i = 0$.

4.31. $2e^z + \sqrt{2} - i\sqrt{2} = 0$.

4.32. $\sin z + 6 = 0$.

4.33. $\mathrm{ch}z + 5i = 0$.

4.34. $2\,\mathrm{ctg}\, z - 3i = 0$.

4.35. $2e^z + \sqrt{3}i - 1 = 0$.

4.16. $5\,\mathrm{th}\,z - 4 + 3i = 0$.

4.17. $7\,\mathrm{tg}\,z + 2\sqrt{3} - 3i = 0$.

4.18. $8\,\mathrm{cth}\,z - 3 - i \cdot 2\sqrt{3} = 0$.

4.19. $\cos z + 5 = 0$.

4.20. $\mathrm{sh}z + 4i = 0$.

4.36. $\mathrm{sh}z + 2 = 0$.

4.37. $\cos z - 3 = 0$.

4.38. $2e^z + \sqrt{3} - i = 0$.

4.39. $\mathrm{cth}\,z - 1 - 2i = 0$.

4.40. $\mathrm{ch}z + 4 = 0$.

Задача 5. Вычертить область, заданную неравенствами.

5.1. $|z-1| \le 1,\ |z+1| > 2$.

5.2. $|z+i| \ge 1,\ |z| < 2$.

5.3. $|z-i| \le 2,\ \mathrm{Re}z > 1$.

5.4. $|z+1| \ge 1,\ |z+i| < 1$.

5.5. $|z+1| < 1,\ |z-i| \le 1$.

5.6. $|z+i| \le 2,\ |z-i| > 2$.

5.7. $|z-1-i| \le 1,\ \mathrm{Im}z > 1,\ \mathrm{Re}z \ge 1$.

5.8. $|z-1+i| \ge 1,\ \mathrm{Re}z < 1, \mathrm{Im}z \le -1$.

5.9. $|z-2-i| \le 2,\ \mathrm{Re}z \ge 3,\ \mathrm{Im}z < 1$.

5.10. $|z-1-i| \ge 1,\ 0 \le \mathrm{Re}z < 2,\ 0 < \mathrm{Im}z \le 2$.

5.11. $|z+i| < 2,\ 0 < \mathrm{Re}z \le 1$.

5.12. $|z-i| \le 1,\ 0 < \mathrm{arg}z < \pi/4$.

5.13. $|z-i| \le 2,\ 0 < \mathrm{Im}z < 2$.

5.14. $|z+i| > 1,\ -\pi/4 \le \mathrm{arg}z < 0$.

5.15. $|z-1-i| < 1,\ |\mathrm{arg}z| \le \pi/4$.

5.16. $|z| < 2,\ -\pi/4 \le \arg(z-1) \le \pi/4.$

5.17. $|z| \le 1,\ \arg(z+i) > \pi/4.$

5.18. $1 < |z-1| \le 2,\ \mathrm{Im}z \ge 0,\ \mathrm{Re}z < 1.$

5.19. $1 \le |z-i| < 2,\ \mathrm{Re}z \le 0, \mathrm{Im}z > 1.$

5.20. $|z| < 2,\ \mathrm{Re}z \ge 1,\ \arg z < \pi/4.$

5.21. $|z| > 1,\ -1 < \mathrm{Im}z \le 1,\ 0 < \mathrm{Re}z \le 2.$

5.22. $|z-1| > 1,\ -1 \le \mathrm{Im}z < 0,\ 0 \le \mathrm{Re}z < 3.$

5.23. $|z+i| < 1,\ -3\pi/4 \le \arg z \le -\pi/4.$

5.24. $|z-i| \le 1,\ -\pi/2 < \arg(z-i) < \pi/4.$

5.25. $z\overline{z} < 2,\ \mathrm{Re}z \le 1, \mathrm{Im}z > -1.$

5.26. $z\overline{z} \le 2,\ \mathrm{Re}z < 1,\ \mathrm{Im}z > -1.$

5.27. $1 < z\overline{z} < 2,\ \mathrm{Re}z > 0,\ 0 \le \mathrm{Im}z \le 1.$

5.28. $|z-1| < 1,\ \arg z \le \pi/4,\ \arg(z-1) > \pi/4.$

5.29. $|z-i| < 1,\ \arg z \ge \pi/4,\ \arg(z+1-i) \le \pi/4.$

5.30. $|z-2-i| \ge 1,\ 1 \le \mathrm{Re}z < 3,\ 0 < \mathrm{Im}z \le 3.$

5.31. $|\mathrm{Re}z| \le 1,\ |\mathrm{Im}z| < 2.$

5.32. $z\overline{z} > 1,\ \mathrm{Re}z > 0,\ \mathrm{Im}z < 0.$

5.33. $2 < |z+1-i| < 3,\ \pi/2 < \arg z < 3\pi/4.$

5.34. $|\mathrm{Re}(z+2)| < 1,\ |\mathrm{Im}(z+2i)| < 2.$

5.35. $\mathrm{Re}z > 1,\ \pi/4 < \arg z < \pi/2.$

5.36. $\mathrm{Im}z < 2,\ |\mathrm{Re}z| > 2.$

5.37. $|\mathrm{Im}z| \le 1,\ \pi/4 < \arg(z+i) < 3\pi/4.$

5.38. $|z| < 2,\ \mathrm{Im}z < 1, 3\pi/4 < \arg z < \pi.$

5.39. $\mathrm{Re}(z-1) \ge 1,\ |\mathrm{Im}z| \le 1.$

5.40 $|\mathrm{Re}(z+i)| < 1,\ \mathrm{Im}(z-i) > 0.$

Задача 6. Найти действительную и мнимую части функции $f(z)$. Найти область ее аналитичности, проверив выполнение условий Коши-Римана. Если функция аналитична, найти ее производную.

6.1. $f(z) = \dfrac{1}{z-i}.$

6.2. $f(z) = \dfrac{1}{z-1-i}.$

6.3. $f(z) = z^2 - iz.$

6.4. $f(z) = iz^2 - 5z + 2i.$

6.5. $f(z) = z^2 + z + 1.$

6.6. $f(z) = iz^3 + 2e^{i\frac{\pi}{6}z}.$

6.7. $f(z) = (1+i)z^2 + iz + i.$

6.8. $f(z) = z^3 - 2e^{i\frac{\pi}{3}z}.$

6.9. $f(z) = \sqrt{2}e^{i\frac{5\pi}{4}}z^3.$

6.10. $f(z) = z^3 + e^{i\frac{5\pi}{6}}.$

6.11. $f(z) = \mathrm{Re}[(z-i)(z+1)].$

6.12. $f(z) = z\mathrm{Re}(zi).$

6.13. $f(z) = |z|\mathrm{Im}z.$

6.14. $f(z) = z^2|z|.$

6.15. $f(z) = |z-1|z.$

6.16. $f(z) = \dfrac{z-2}{z-i}.$

6.21. $f(z) = \ln|z|.$

6.22. $f(z) = \ln|z-i|.$

6.23. $f(z) = z + \ln|z|.$

6.24. $f(z) = \overline{z} + \ln|z|.$

6.25. $f(z) = \overline{z}|z|.$

6.26. $f(z) = \dfrac{1}{\overline{z}-i}.$

6.27. $f(z) = \overline{z}\mathrm{Re}(iz).$

6.28. $f(z) = (\overline{z})^2\mathrm{Im}z.$

6.29. $f(z) = (\overline{z})^2.$

6.30. $f(z) = e^{i\overline{z}}.$

6.31. $f(z) = (z)^2\mathrm{Re}z.$

6.32. $f(z) = z\mathrm{Re}z^2.$

6.33. $f(z) = z^2 + e^{iz}.$

6.34. $f(z) = \dfrac{1}{z-2+3i}.$

6.35. $f(z) = z\cos z.$

6.36. $f(z) = iz^2 + \cos iz.$

6.17. $f(z) = e^{z^2}$.

6.18. $f(z) = e^{|z|^2}$.

6.19. $f(z) = \sin|z|$.

6.20. $f(z) = \sqrt{2}e^{i\frac{3\pi}{4}}\cos|z|$.

6.37. $f(z) = \sin|z|e^{3\pi i}$.

6.38. $f(z) = (z)^2\mathrm{Re}(iz)$.

6.39. $f(z) = \dfrac{1}{\overline{z}+1+i}$.

6.40. $f(z) = z^2\mathrm{Im}z$.

Задача 7. Восстановить аналитическую в окрестности точки z_0 функцию $f(z)$ по известной действительной части $u(x,y)$ или мнимой части $v(x,y)$ и значению $f(z_0)$.

7.1. $v(x,y) = x^2 - y^2 - 2y;\ f(0) = 0.$

7.2. $u(x,y) = x^2 - y^2 - 2y;\ f(0) = 0.$

7.3. $v(x,y) = xe^x\cos y - e^x y\sin y;\ f(0) = 0.$

7.4. $u(x,y) = \dfrac{x}{x^2+y^2};\ f(1) = 1.$

7.5. $u(x,y) = x^3 - 3xy^2;\ f(0) = 0.$

7.6. $u(x,y) = \mathrm{sh}x\cos y;\ f(0) = 0.$

7.7. $u(x,y) = \dfrac{x}{x^2+y^2} + x;\ f(1) = 2.$

7.8. $v(x,y) = 3x^2y - y^3;\ f(0) = 0.$

7.9. $v(x,y) = 4xy - 3y;\ f(0) = 4.$

7.10. $v(x,y) = \mathrm{sh}x\sin y;\ f(0) = 1.$

7.11. $u(x,y) = x\sin x\mathrm{ch}y - y\cos x\mathrm{sh}y;\ f(0) = 0.$

7.12. $v(x,y) = 2x^2 - 2y^2 + 3x;\ f(0) = 0.$

7.13. $v(x,y) = e^x\cos y + y;\ f(0) = 0.$

7.14. $u(x,y) = x - e^{-x}\cos y;\ f(0) = -1.$

7.15. $v(x,y) = \cos y \mathrm{ch} x;\ f(0) = 1.$

7.16. $u(x,y) = x^3 - 3xy^2;\ f(0) = 0.$

7.17. $v(x,y) = y - \dfrac{y}{x^2+y^2};\ f(1) = 2.$

7.18. $u(x,y) = 2x^2 - 2y^2 - 3x + 4;\ f(0) = 4.$

7.19. $v(x,y) = \mathrm{ch} x \sin y;\ f(0) = 0.$

7.20. $v(x,y) = e^x y \cos y - x e^x \sin y;\ f(0) = 0.$

7.21. $v(x,y) = 2xy + 2x;\ f(0) = 0.$

7.22. $u(x,y) = -2xy - 2x;\ f(0) = 0.$

7.23. $v(x,y) = y \sin x \mathrm{ch} y + x \cos x \mathrm{sh} y;\ f(0) = 0.$

7.24. $u(x,y) = x - e^x \sin y;\ f(0) = 0.$

7.25. $v(x,y) = x^3 - 3xy^2;\ f(0) = 1.$

7.26. $u(x,y) = -2xy - 2y;\ f(0) = i.$

7.27. $u(x,y) = -4xy - 3y;\ f(0) = 0.$

7.28. $u(x,y) = e^x \cos y;\ f(0) = 2 + i.$

7.29. $u(x,y) = x^2 - y^2 + 1;\ f(0) = 1.$

7.30. $v(x,y) = \mathrm{ch} x \cos y;\ f(0) = 1.$

7.31. $u(x,y) = \frac{x}{x^2+y^2} + 2x;\ f(1) = 3.$

7.32. $u(x,y) = x^3 - 3xy^2 - x;\ f(0) = 0.$

7.33. $v(x,y) = \dfrac{-y}{x^2+y^2} + 2y;\ f(1) = 3.$

7.34. $v(x,y) = 2xy - 2y;\ f(0) = 1.$ 14

7.35. $v(x,y) = e^{-y} \cos x + x;\ f(0) = 1.$

7.36. $u(x,y) = 2xy;\ f(0) = 1.$

7.37. $u(x,y) = -\mathrm{sh}x \sin y;\ f(0) = 1.$

7.38. $v(x,y) = y + e^{-x} \sin y;\ f(0) = 1.$

7.39. $u(x,y) = 1 - e^{x} \sin y;\ f(0) = 1 + i.$

7.40. $u(x,y) = -3x^2y + y^3 + 1;\ f(0) = 1.$

Задача 8. Вычислить интеграл от функции комплексной переменной. (Применение формулы Ньютона-Лейбница обосновать проверкой условий Коши-Римана).

8.1. $\int\limits_C (z+1)e^z dz;\ C : \{|z| = 1,\ \mathrm{Re}\ z \geqslant 1\}.$

8.2. $\int\limits_{ABC} z^2 e^{z^3} dz;\ ABC -$ ломаная : $\{z_A = i, z_B = 1 + i, z_C = 0\}.$

8.3. $\int\limits_C (z + e^z) dz;\ C : \{|z| = 2, \mathrm{Re}\ z \geqslant 0, \mathrm{Im}\ z \geqslant 0\}.$

8.4. $\int\limits_{ABC} (z-1)e^{(z-1)^2} dz;$

$ABC -$ ломаная : $\{z_A = 1 - i, z_B = 1 + i, z_C = 2\}.$

8.5. $\int\limits_{ABCD} (z^5 + z\cos z) dz;$

$ABCD -$ ломаная : $\{z_A = i - 1, z_B = i + 1, z_C = 1 - i, z_D = -1 + i\}.$

8.6. $\int\limits_{ABC} (z^2 + 7z + 1) dz;\ ABC -$ ломаная : $\{z_A = 1, z_B = 0, z_C = 1 - i\}.$

8.7. $\int\limits_C (\cos z + z^4 + e^z) dz;\ C : \{|z + 1| = 1.5\}.$

8.8. $\int\limits_{ABC} (z+i)e^{(z+i)^2} dz;\ ABC -$ ломаная : $\{z_A = -2, z_B = 0, z_C = 1 - i\}.$

8.9. $\int\limits_C (z^2 e^{z^3} + 1)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0, \mathrm{Im}\ z \geqslant 0\}.$

8.10. $\int\limits_{ABC} (z+1)^2 e^{(z+1)^3} dz;$

ABC – ломаная : $\{z_A = i - 1, z_B = i, z_C = -1\}.$

8.11. $\int\limits_{AB} (12z^5 + 4z^3 + 1)dz;\ AB$ – отрезок прямой : $\{z_A = 1, z_B = i\}.$

8.12. $\int\limits_{ABC} z^3 e^{z^4} dz;\ ABC$ – ломаная : $\{z_A = i, z_B = 1, z_C = 0\}.$

8.13. $\int\limits_{ABC} (z^2 + \cos z)dz;\ ABC$ – ломаная : $\{z_A = 0, z_B = 1, z_C = i\}.$

8.14. $\int\limits_{ABC} (z-i)^2 e^{(z-i)^3} dz;$

ABC – ломаная : $\{z_A = 2i, z_B = 1 + 2i, z_C = i\}.$

8.15. $\int\limits_C (\sin z + z^3 + 1)dz;\ C : \{|z + i| = 2\}.$

8.16. $\int\limits_C (\cos iz + 3z^2)dz;\ C : \{|z| = 1, \mathrm{Im}\ z \geqslant 0\}.$

8.17. $\int\limits_C (\mathrm{ch} z + z)dz;\ C : \{|z| = 1, \mathrm{Im}\ z \leqslant 0\}.$

8.18. $\int\limits_{AB} (3z^2 + 2z)dz;\ AB : \{y = x^2, z_A = 0, z_B = 1 + i\}.$

8.19. $\int\limits_C (\sin iz + z)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0\}.$

8.20. $\int\limits_{ABC} (z^2 + 1)dz;\ ABC$ – ломаная : $\{z_A = 0, z_B = -1 + i, z_C = i\}.$

8.21. $\int\limits_{AB} (1 + 2z)dz;\ AB : \{y = x^3, z_A = 0, z_B = 1 + i\}.$

8.22. $\int\limits_{ABC} (\mathrm{ch} z + \cos(iz))dz;\ ABC$ – ломаная : $\{z_A = 0, z_B = -1, z_C = i\}.$

8.23. $\int\limits_C (z^3 e^{z^4} - i)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0\}.$

8.24. $\int\limits_{ABC} (z^9 + 1)dz;\ ABC -$ ломаная $: \{z_A = 0, z_B = 1 + i, z_C = i\}.$

8.25. $\int\limits_C (ze^{z^2} + 1)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0, \mathrm{Im}\ z \geqslant 0\}.$

8.26. $\int\limits_{ABC} (z-1)^2 e^{(z-1)^3} dz;$

$ABC -$ ломаная $: \{z_A = 1 + i, z_B = i + 2, z_C = 1\}.$

8.27. $\int\limits_{ABC} z\cos z dz;\ ABC -$ ломаная $: \{z_A = 0, z_B = 1 + i, z_C = 1\}.$

8.28. $\int\limits_{ABC} (z^5 + \sin z)dz;\ ABC -$ ломаная $: \{z_A = 0, z_B = 1, z_C = 2i\}.$

8.29. $\int\limits_{ABCD} (e^z + z\sin z)dz;$

$ABCD -$ ломаная $: \{z_A = i, z_B = 1, z_C = -1, z_D = i\}.$

8.30. $\int\limits_{ABC} z\sin z dz;\ ABC -$ ломаная $: \{z_A = 0, z_B = 1 + i, z_C = 1\}.$

8.31. $\int\limits_C (\sin z + z^3)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0\}.$

8.32. $\int\limits_C (3z + \mathrm{sh}z)e^z dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0\}.$

8.33. $\int\limits_C (\cos z + z^4)dz;\ C : \{|z| = 1, \mathrm{Im}\ z \geqslant 0\}.$

8.34. $\int\limits_C (3 + 2z + z^4)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \leqslant 0\}.$

8.35. $\int\limits_C (\mathrm{sh}5z + 4z)dz;\ C : \{|z| = 2, \mathrm{Re}\ z \geqslant 0\}.$

8.36. $\int\limits_{AB} (z^2 + 2\cos z)dz;\ AB : \{y = x^3, z_A = 0, z_B = 1 + i\}.$

8.37. $\int\limits_C (\sin(3z) + z^4)dz;\ C : \{|z| = 1, \mathrm{Re}\ z \geqslant 0\, \mathrm{Im}\ z \geqslant 0\}.$

8.38. $\int\limits_{ABC} (z^3 + \sin 3z)dz;$

$ABC -$ ломаная : $\{z_A = 0, z_B = 1 + i, z_C = i + 2\}.$

8.39. $\int\limits_C (\cos 3iz + \mathrm{sh}z)dz;\ C : \{|z| = 1, \mathrm{Im}\ z \leqslant 0\}.$

8.40. $\int\limits_{ABC} (z + ze^z)dz;\ ABC -$ ломаная : $\{z_A = 0, z_B = 2 + i, z_C = 2\}.$

Задача 9. Вычислить интеграл.

9.1. $\int\limits_{AB} (\overline{z})^2 dz;\ AB : \{y = x^2, z_A = 0, z_B = 1 + i\}.$

9.2. $\int\limits_{AB} \mathrm{Im}\ z^3 dz;\ AB -$ отрезок прямой : $\{z_A = 0, z_B = 2 + 2i\}.$

9.3. $\int\limits_{ABC} |z|dz;\ ABC -$ ломаная : $\{z_A = 0, z_B = -1 + i, z_C = 1 + i\}.$

9.4. $\int\limits_{AB} (\overline{z})^2 dz;\ ABC -$ отрезок прямой : $\{z_A = 0, z_B = 1 + i\}.$

9.5. $\int\limits_C z|z|dz;\ C : \{|z| = 1, \mathrm{Im}\ z \geqslant 0\}.$

9.6. $\int\limits_{ABC} \mathrm{Re}\ \dfrac{\overline{z}}{z} dz;\ AB : \{|z| = 1, \mathrm{Im}\ z \geqslant 0\},\ BC -$ отрезок$\{z_B = 1, z_C = 2\}.$

9.7. $\int\limits_{ABC} (z + \overline{z})^4 dz;\ ABC -$ ломаная : $\{z_A = 0, z_B = 1, z_C = 1 - i\}.$

9.8. $\int\limits_C z^2 \overline{z} dz;\ C : \{|z| = \sqrt{5}, \mathrm{Im}\ z \geqslant 0, \mathrm{Re}\ z \geqslant 0\}.$

9.9. $\int\limits_{AB} e^{|z|^3} \mathrm{Im}\ z dz;\ AB -$ отрезок прямой : $\{z_B = 0, z_A = 1 + i\}.$

9.10. $\int\limits_C \overline{z}|\overline{z}|^2 dz;\ C : \{|z| = \sqrt[4]{4}, 3\pi/4 < \arg z < \pi\}$

9.11. $\int\limits_{ABC} (z - \overline{z})^2 dz;\ ABC -$ ломаная $: \{z_A = -1, z_B = 0, z_C = i\}.$

9.12. $\int\limits_C \overline{z}|z| dz;\ C : \{|z| = 4, \mathrm{Re}\ z \geqslant 0\}.$

9.13. $\int\limits_C |z| \mathrm{Re}\ z^2 dz;\ C : \{|z| = R, \mathrm{Im}\ z \geqslant 0\}.$

9.14. $\int\limits_{ABC} (z + \overline{z}) dz;\ ABC -$ ломаная $: \{z_A = -1, z_B = i, z_C = 1\}.$

9.15. $\int\limits_C \overline{z}|z|^3 dz;\ C : \{|z| = \sqrt[5]{5}, 0 < \arg z < 3\pi/4\}.$

9.16. $\int\limits_C z^2|z| dz;\ C : \{|z| = R, \mathrm{Im}\ z \geqslant 0\}.$

9.17. $\int\limits_C z(\overline{z})^2 dz;\ C : \{|z| = \sqrt{2}, \mathrm{Im}\ z \geqslant 0\}.$

9.18. $\int\limits_{ABC} (|z| + \mathrm{Re}\ z) dz;$

$ABC -$ ломаная $: \{z_A = -1 + i, z_B = 0, z_C = 1 + i\}.$

9.19. $\int\limits_{ABC} z\overline{z} dz;\ AB : \{|z| = 1, \mathrm{Re}\ z \geqslant 0, \mathrm{Im}\ z \geqslant 0\},$

$BC -$ отрезок$\{z_B = 1, z_C = 0\}.$

9.20. $\int\limits_{AB} z\mathrm{Re}\ z^2 dz;\ AB -$ отрезок прямой $: \{z_A = 0, z_B = 1 + 2i\}.$

9.21. $\int\limits_C z^2(\overline{z})^5 dz;\ C : \{|z| = 1, \mathrm{Im}\ z \geqslant 0, \mathrm{Re}\ z \geqslant 0\}.$

9.22. $\int\limits_C (|z|^3 + \sqrt{3}) dz;\ C : \{|z| = \sqrt{3}, \pi/3 < \arg z < 2\pi/3\}.$

9.23. $\int\limits_{ABC}(z-\overline{z})dz;\ ABC-$ ломаная : $\{z_A=-1, z_B=i, z_C=1\}$.

9.24. $\int\limits_{C}(|\overline{z}|^2+1)dz;\ C:\{|z|=2, \pi/8<\arg z<3\pi/8\}$.

9.25. $\int\limits_{ABC}(|z|+\mathrm{Im}\ z)dz;$

$ABC-$ ломаная : $\{z_A=-1+i, z_B=0, z_C=1+i\}$.

9.26. $\int\limits_{C}|\overline{z}|^3dz;\ C:\{|z|=2, \pi/4<\arg z<3\pi/4\}$.

9.27. $\int\limits_{C}\frac{z}{\overline{z}}dz;\ C-$ граница области : $\{1<|z|<2, \mathrm{Re}\ z>0\}$.

9.28. $\frac{1}{2i}\int\limits_{C}|z|dz;\ C:\{|z|=R\}$.

9.29. $\int\limits_{C}|z|dz;\ C:\{|z|=\sqrt{2}, 3\pi/4<\arg z<5\pi/4\}$.

9.30. $\int\limits_{AB}z\mathrm{Im}\ z^2dz;\ AB-$ отрезок прямой : $\{z_A=0, z_B=1+i\}$.

9.31. $\int\limits_{C}|z|^2(\overline{z})^3dz;\ C:\{|z|=1, 0<\arg z<\pi/2\}$.

9.32. $\int\limits_{C}\overline{z}|z|dz;\ C:\{|z|=9, \mathrm{Re}\ z\leqslant 0\}$.

9.33. $\int\limits_{AB}z^2\mathrm{Im}\ zdz;\ AB-$ отрезок прямой : $\{z_B=1+i, z_A=1\}$.

9.34. $\int\limits_{ABC}|\overline{z}|^2dz;\ AB-$ отрезок прямой : $\{z_B=1+i, z_A=0\}$.

9.35. $\int\limits_{ABC}(\overline{z}-2z)dz;\ ABC-$ ломаная : $\{z_A=-1, z_B=i, z_C=1\}$.

9.36. $\int\limits_{C}z^2|\overline{z}|^5dz;\ C:\{|z|=5, \mathrm{Re}\ z\geqslant 0, \mathrm{Im}\ z\geqslant 0\}$.

9.37. $\int\limits_C \dfrac{\overline{z}}{z} dz;\ C -$ граница области : $\{|z| < 1, \pi/4 < \arg z < \pi/2\}$.

9.38. $\int\limits_{AB} e^{|z|^2} dz;\ AB -$ отрезок прямой : $\{z_B = 1 + i, z_A = 0\}$.

9.39. $\int\limits_C z|z|dz;\ C : \{|z| = 1, \operatorname{Im}\ z \geqslant 0\}$.

9.40. $\int\limits_{ABC} |z|\operatorname{Im}\ z^2 dz;\ ABC -$ ломаная : $\{z_A = 0, z_B = 1 + i, z_C = 2 + i\}$.

Задача 10. Вычислить интеграл.

10.1. $\oint\limits_{|z|=0.5} \dfrac{dz}{z(z^2+1)}$.

10.2. $\oint\limits_{|z-1-i|=1.25} \dfrac{dz}{z^2(z-1)}$.

10.3. $\oint\limits_{|z-i|=1.5} \dfrac{dz}{z(z^2+4)}$.

10.4. $\oint\limits_{|z|=1} \dfrac{2+\sin z}{z(z+2i)} dz$.

10.5. $\oint\limits_{|z-1.5|=2} \dfrac{e^z}{z-\pi} dz$.

10.6. $\oint\limits_{|z-1.5|=2} \dfrac{z(\sin z+2)}{z-\pi} dz$.

10.7. $\oint\limits_{|z-1|=3} \dfrac{ze^z}{z-\pi} dz$.

10.8. $\oint\limits_{|z-1.5|=2} \dfrac{2z(z-1)}{z-\pi} dz$.

10.9. $\oint\limits_{|z-0.25|=0.5} \dfrac{z(z+1)^2}{z-0.5} dz$.

10.21. $\oint\limits_{|z|=\pi/2} \dfrac{(z^2+z+3)}{z(z+\pi)} dz$.

10.22. $\oint\limits_{|z|=0.5} \dfrac{(z^3-i)}{2z(z-\pi)} dz$.

10.23. $\oint\limits_{|z-1|=2} \dfrac{z(z+\pi)}{z-\pi/2} dz$.

10.24. $\oint\limits_{|z|=2} \dfrac{z^2+\sin z+2}{z^2+\pi z} dz$.

10.25. $\oint\limits_{|z-1.5|=1} \dfrac{z(z+\pi)}{(z-\pi/3)(z-\pi)} dz$.

10.26. $\oint\limits_{|z-1.5|=1} \dfrac{\sin z}{z(z+\pi/3)(z-\pi)} dz$.

10.27. $\oint\limits_{|z-\pi|=0.5} \dfrac{(z^2+\pi)^2}{i(z-\pi)} dz$.

10.28. $\oint\limits_{|z|=2} \dfrac{\sin^2 z}{z(z-\pi/2)} dz$.

10.29. $\oint\limits_{|z-\pi|=2} \dfrac{\cos^2 z}{z(z-\pi)} dz$.

10.10. $\oint\limits_{|z-0.5|=1} \dfrac{\cos(iz)(z-1)}{\pi z}dz.$

10.11. $\oint\limits_{|z-3|=1} \dfrac{\sin 3z+2}{(z-\pi)z^2}dz$

10.12. $\oint\limits_{|z-0.25|=0.5} \dfrac{z(z+1)^2}{z-0.5}dz.$

10.13. $\oint\limits_{|z|=0.5} \dfrac{e^{zi}+2}{3zi}dz.$

10.14. $\oint\limits_{|z-2|=3} \dfrac{\cos^2 z+1}{z^2-\pi^2}dz.$

10.15. $\oint\limits_{|z+1|=2} \dfrac{\cos z-2+z^2}{z^2+3\pi z}dz.$

10.16. $\oint\limits_{|z-6|=1} \dfrac{\sin^3 z+2}{z^2-4\pi^2}dz.$

10.17. $\oint\limits_{|z+1|=0.5} \dfrac{\mathrm{tg}z+2}{4z^2+\pi z}dz.$

10.18. $\oint\limits_{|z+1.5|=1} \dfrac{\cos^2 z+3}{2z^2+\pi z}dz.$

10.19. $\oint\limits_{|z+1|=2} \dfrac{\sin^2 z-3}{z^2+2\pi z}dz.$

10.20. $\oint\limits_{|z|=0.25} \dfrac{\ln(e+z)}{z(z+\pi/4)}dz.$

10.30. $\oint\limits_{|z-3|=2} \dfrac{z^3+\sin 2z}{z(z-\pi)}dz.$

10.31. $\oint\limits_{|z-1|=2} \dfrac{z^2+1}{z(z^2+4)}dz.$

10.32. $\oint\limits_{|z+2|=3} \dfrac{\cos^2 z+1}{z^2-\pi^2}dz.$

10.33. $\oint\limits_{|z|=1} \dfrac{\sin^2 z}{z\cos z}dz.$

10.34. $\oint\limits_{|z|=1} \dfrac{\cos z-2+z^2}{z^2+\pi z}dz.$

10.35. $\oint\limits_{|z|=1} \dfrac{\sin^2 z-3}{z^2+\pi z}dz.$

10.36. $\oint\limits_{|z+1|=0.5} \dfrac{\ln z+3}{z(z+1)}dz.$

10.37. $\oint\limits_{|z|=1} \dfrac{e^z+3z}{z(z+2)}dz.$

10.38. $\oint\limits_{|z+3.5|=1} \dfrac{z+\sin^2 z}{z^2+3.5z}dz.$

10.39. $\oint\limits_{|z-0.5|=1} \dfrac{\sin^3 z+1}{z(z+5)}dz.$

10.40. $\oint\limits_{|z|=4} \dfrac{ze^z+\cos z}{z(z-8)}dz.$

Задача 11. Вычислить интеграл.

11.1. $\oint\limits_{|z|=1} \dfrac{\cos z^2-1}{z^3}dz.$

11.2. $\oint\limits_{|z|=1/2} \dfrac{2-z^2+3z^3}{4z^3}dz.$

10.21. $\oint\limits_{|z|=1} \dfrac{\cos z^2-1}{z^4}dz.$

11.22. $\oint\limits_{|z|=1/2} \dfrac{2-5z^4+3z^3}{4z^5}dz.$

11.3. $\oint\limits_{|z|=1} \frac{e^z+1}{z^2}dz.$

11.4. $\oint\limits_{|z|=2} \frac{\sin z^3 dz}{z^2}.$

11.5. $\oint\limits_{|z|=1/3} \frac{1-2z+3z^2+4z^3}{2z^3}dz.$

11.6. $\oint\limits_{|z|=2} \frac{1-\cos z^2}{z^2}dz.$

11.7. $\oint\limits_{|z|=1} \frac{5-2z^3+3z^4}{z^4}dz.$

11.8. $\oint\limits_{|z|=3} \frac{1+\sin z}{z^2}dz.$

11.9. $\oint\limits_{|z|=1/2} \frac{e^{2z^2-1}}{z^3}dz.$

11.10. $\oint\limits_{|z|=1/3} \frac{3-2z^2+4z^4}{z^3}dz.$

11.11. $\oint\limits_{|z|=2} \frac{z-2\sin z}{3z^4}dz.$

11.12. $\oint\limits_{|z|=1} \frac{1-3z^2+z^3}{z^4}dz.$

11.13. $\oint\limits_{|z|=1/2} \frac{1-3z^3+4z^5}{4z^6}dz.$

11.14. $\oint\limits_{|z|=2} \frac{e^{2z}-z}{z^2}dz.$

11.15. $\oint\limits_{|z|=1} \frac{\cos iz-1}{z^3}dz.$

11.16. $\oint\limits_{|z|=1} \frac{\cos iz-1}{z^5}dz.$

11.23. $\oint\limits_{|z|=1} \frac{ze^z-z-1}{z^3}dz.$

11.24. $\oint\limits_{|z|=2} \frac{\sin iz^2}{z^3}dz.$

11.25. $\oint\limits_{|z|=1/2} \frac{2-3z^5+z^4}{4z^6}dz.$

10.26. $\oint\limits_{|z|=1} \frac{e^{iz}-1}{z^3}dz.$

11.27. $\oint\limits_{|z|=1/3} \frac{1-z^4+3z^6}{2z^3}dz.$

11.28. $\oint\limits_{|z|=2} \frac{\cos 2iz}{z^3}dz$

11.29. $\oint\limits_{|z|=1/3} \frac{e^z-\sin z}{z^2}dz.$

11.30. $\oint\limits_{|z|=3} \frac{3z^2+2z^3-2}{4z^5}dz.$

11.31. $\oint\limits_{|z|=1} \frac{e^{z^2-1}}{z^4}dz.$

11.32. $\oint\limits_{|z|=2} \frac{1-\cos z^2}{z^5}dz.$

11.33. $\oint\limits_{|z|=1} \frac{\cos z+1}{z^3}dz.$

11.34. $\oint\limits_{|z|=1/2} \frac{z^5+2z^2+4z+1}{z^3}dz.$

11.35. $\oint\limits_{|z|=1/2} \frac{\cos z+\sin 3z}{z^4}dz.$

11.36. $\oint\limits_{|z|=1} \frac{\cos z+z^3}{z^4}dz.$

11.17. $\oint\limits_{|z|=1/3} \dfrac{1-2z^4+3z^5}{z^4}dz.$

11.18. $\oint\limits_{|z|=3} \dfrac{z^2+\cos z}{z^3}dz.$

11.19. $\oint\limits_{|z|=1/2} \dfrac{z^5-3z^3+5z}{z^4}dz.$

11.20. $\oint\limits_{|z|=2} \dfrac{z-\sin z}{z^4}dz.$

11.37. $\oint\limits_{|z|=1} \dfrac{\sin z+z^2}{z^3}dz.$

11.38. $\oint\limits_{|z|=1} \dfrac{\sin z+z^2}{z^3}dz.$

11.39. $\oint\limits_{|z|=2} \dfrac{\cos z+z}{z^4}dz.$

11.40. $\oint\limits_{|z|=1} \dfrac{7\cos z+z^3}{z^4}dz.$

Задача 12. Для данной функции $f(z)$ найти изолированные особые точки и определить их тип. Найти радиус круга сходимости ряда Тейлора для заданной функции $f(z)$ с центром в точке z_0.

12.1. $f(z)=\dfrac{(z^2-1)\sin(3z-1)}{z^2-2z-8},\ z_0=2i.$

12.2. $f(z)=\dfrac{(z^2-3iz+i)\cos(2\pi z-6)}{\mathrm{ch}^2(\pi z)},\ z_0=1+4.5i.$

12.3. $f(z)=\dfrac{(e^{2iz}+2)\mathrm{ch}(iz-3\pi-1)}{z^3-6iz^2-z},\ z_0=-3-5i.$

12.4. $f(z)=\dfrac{\mathrm{sh}(3z+i)(e^{2iz}-e^z)}{\sin\left(\dfrac{z\pi i}{2}\right)},\ z_0=4i-3.$

12.5. $f(z)=\dfrac{\cos(z^3-iz)\sin(2-z^6)}{e^{12\pi z}-e^{10\pi z}},\ z_0=6i-7.$

12.6. $f(z)=\dfrac{(z^2-3i)\mathrm{sh}(-6z-1+\pi i)}{z^2-6iz-2},\ z_0=2i+8.$

12.7. $f(z)=\dfrac{\sin(z^2-i-z)}{\cos^2\left(\dfrac{\pi z}{2}\right)},\ z_0=4-i.$

12.8. $f(z) = \dfrac{z^3 \cos(\pi z - 6i)}{z^2 - iz - 2},\ z_0 = i - 6.$

12.9. $f(z) = \dfrac{(z^2 - 1)(e^{2z+i} - 2)}{\mathrm{sh}^3(3\pi z)},\ z_0 = 2 - 7i.$

12.10. $f(z) = \dfrac{\sin(z^2 + 1)\cos(1 + e^z)}{z^3 - 7iz - 7z^2},\ z_0 = 4i - 2.$

12.11. $f(z) = \dfrac{z^2 \cos(z - 3) - z^3 \sin^2(z + i)}{\sin^2(3\pi z)},\ z_0 = 2i + 2.$

12.12. $f(z) = \dfrac{\mathrm{sh}(z^7 - 1)\sin(iz - 7)}{e^{3.5\pi z} - e^{3\pi z}},\ z_0 = 2 - 5i.$

12.13. $f(z) = \dfrac{\cos(z^2 - 2zi)\cos(3z - i)}{\mathrm{sh}\left(\dfrac{\pi z}{2}\right)},\ z_0 = 3 - 3i.$

12.14. $f(z) = \dfrac{\sin(z^2 - i - 2z)(e^{z+3} - 2)}{\cos(\pi i z)},\ z_0 = 9 - 2.5i.$

12.15. $f(z) = \dfrac{\mathrm{sh}(z^3 - iz)e^{iz}}{z^3 - iz^2 + z},\ z_0 = 2 - 2i.$

12.16. $f(z) = \dfrac{(z^2 - 6iz + 2)\sin(z^2 - 6iz - 1)}{e^{2.5i\pi z} - e^{2i\pi z}},\ z_0 = 7 - i.$

12.17. $f(z) = \dfrac{z^2 + \cos(3i - z)e^z}{\mathrm{sh}\left(\dfrac{z\pi i}{6}\right)},\ z_0 = -4 - i.$

12.18. $f(z) = \dfrac{\cos z \sin(4iz^2 - 5z)}{z^3 - 4iz^2 - 5z},\ z_0 = 5 - i.$

12.19. $f(z) = \dfrac{\cos(2z^2 - 1)(z^3 - z^2 - 6i)}{\sin\left(\dfrac{\pi z i}{2}\right)},\ z_0 = 6i - 1.$

12.20. $f(z) = \dfrac{\operatorname{sh}(zi - z^3 - 2)e^{2zi+3}}{e^{36\pi z} - e^{4\pi z}},\ z_0 = 6 - 3i.$

12.21. $f(z) = \dfrac{\sin(\pi z - 1)e^{3z+2}(z + i)}{\operatorname{ch}\left(\dfrac{zi\pi}{2}\right)},\ z_0 = 4i - 6.$

12.22. $f(z) = \dfrac{\operatorname{ch}(2z^3 - \pi iz)e^{3z}}{z^3 - z^2 + 9z - 9},\ z_0 = -1 - i.$

12.23. $f(z) = \dfrac{\operatorname{sh}(3z^3 + z - 2i)e^{2iz}}{\cos(\pi iz - 4.5\pi)},\ z_0 = 4 - 3i.$

12.24. $f(z) = \dfrac{\cos(z^3 - 2)(e^{2z} - e^z)}{z^2 - 18i - 3z - 6iz},\ z_0 = -2 + i.$

12.25. $f(z) = \dfrac{\ln(1 - 3z)\ln(2 - z)}{\sin^2(\pi zi)},\ z_0 = 3 - 5i.$

12.26. $f(z) = \dfrac{\cos(z^2 + z - i)e^{2iz}}{\operatorname{sh}\left(\dfrac{z\pi}{4}\right)},\ z_0 = 6i - 2.$

12.27. $f(z) = \dfrac{e^{2iz}(e^{6iz} - 4)}{z^3 - 3iz^2 - z},\ z_0 = 4i + 4.$

12.28. $f(z) = \dfrac{\cos(z - i)\sin(2z - 7i)}{\cos\left(\dfrac{zi\pi}{4}\right)},\ z_0 = -5i + 5.$

12.29. $f(z) = \dfrac{(z^3 + 1 + 6z)(e^{z^2} - 1)}{\sin(2\pi z)},\ z_0 = 1 + 6i.$

12.30. $f(z) = \dfrac{e^{2z}\sin^2(z - \pi iz)}{z^3 + 4iz^2 + 5z},\ z_0 = 5 + 2i.$

12.31. $f(z) = \dfrac{\operatorname{ch}(z^3 + iz - 2)\cos(iz - \pi i + 3)}{e^{4i\pi z} - e^{3i\pi z}},\ z_0 = 7i + 5.$

12.32. $f(z) = f(z) = \dfrac{(z^2+1)\cos(3z-1)}{z^2-2iz+8},\ z_0 = 2+i.$

12.33. $f(z) = \dfrac{e^{iz}\sin(z+i\pi)}{e^{7\pi z}-e^{2\pi z}},\ z_0 = -3+2i.$

12.34. $f(z) = \dfrac{\sin(z^2+iz)}{\cos(\pi iz)},\ z_0 = 2+6i.$

12.35. $f(z) = \dfrac{\cos(z-i)\sin z}{\cos(\pi z)},\ z_0 = i+4.$

12.36. $f(z) = \dfrac{\mathrm{sh}(z^2+iz)}{e^{2i\pi z}-e^{i\pi z}},\ z_0 = 3+i.$

12.37. $f(z) = \dfrac{(z^2+1)\sin(3z-1)}{z^2-3iz+7},\ z_0 = i-6.$

12.38. $f(z) = \dfrac{(e^{\pi iz}-1)\cos z^2}{z^3+3iz^2-7z},\ z_0 = 1-6i.$

12.39. $f(z) = \dfrac{\mathrm{sh}z e^{iz}}{\sin^2(\pi z)},\ z_0 = 5-2i.$

12.40. $f(z) = \dfrac{\mathrm{sh}(3z+i)(e^{2iz}-1)}{\sin(2\pi z)},\ z_0 = 2+i.$

Задача 13. Найти все лорановские разложения функции $f(z)$ по степеням $z - z_0$.

13.1. $f(z) = \dfrac{z+1}{z(z-1)},\ z_0 = 1+2i.$

13.2. $f(z) = \dfrac{z+1}{z(z-1)},\ z_0 = -3-2i.$

13.3. $f(z) = \dfrac{z+1}{z(z-1)},\ z_0 = 2-3i.$

13.4. $f(z) = \dfrac{z+1}{z(z-1)},\ z_0 = -2-i.$

13.5. $f(z) = \dfrac{z-1}{z(z+1)},\ z_0 = 1+3i.$

13.6. $f(z) = \dfrac{z-1}{z(z+1)},\ z_0 = 2-i.$

13.7. $f(z) = \dfrac{z-1}{z(z+1)},\ z_0 = -1+2i.$

13.8. $f(z) = \dfrac{z-1}{z(z+1)},\ z_0 = -2-3i.$

13.9. $f(z) = \dfrac{z+3}{z^2-1},\ z_0 = 2+i.$

13.10. $f(z) = \dfrac{z+3}{z^2-1},\ z_0 = 3-i.$

13.11. $f(z) = \dfrac{z+3}{z^2-1},\ z_0 = -2+3i.$

13.12. $f(z) = \dfrac{z+3}{z^2-1},\ z_0 = -2-2i.$

13.13. $f(z) = \dfrac{z}{z^2+1},\ z_0 = 2+i.$

13.14. $f(z) = \dfrac{z}{z^2+1},\ z_0 = 1-2i.$

13.15. $f(z) = \dfrac{z}{z^2+1},\ z_0 = -3+i.$

13.16. $f(z) = \dfrac{z}{z^2+1},\ z_0 = -3-2i.$

13.17. $f(z) = 4\dfrac{z+2}{(z-1)(z+3)},\ z_0 = -2+2i.$

13.18. $f(z) = 4\dfrac{z+2}{(z-1)(z+3)},\ z_0 = 1-3i.$

13.19. $f(z) = 4\dfrac{z+2}{(z-1)(z+3)},\ z_0 = -3-i.$

13.20. $f(z) = 4\dfrac{z+2}{(z-1)(z+3)},\ z_0 = -2+i.$

13.21. $f(z) = 4\dfrac{z-2}{(z+1)(z-3)},\ z_0 = -1-2i.$

13.22. $f(z) = 4\dfrac{z-2}{(z+1)(z-3)},\ z_0 = 3+i.$

13.23. $f(z) = 4\dfrac{z-2}{(z+1)(z-3)},\ z_0 = 2-2i.$

13.24. $f(z) = 4\dfrac{z-2}{(z+1)(z-3)},\ z_0 = -2-i.$

13.25. $f(z) = \dfrac{2z}{z^2+4},\ z_0 = -1-3i.$

13.26. $f(z) = \dfrac{2z}{z^2+4},\ z_0 = -3+2i.$

13.27. $f(z) = \dfrac{2z}{z^2+4},\ z_0 = 2+3i.$

13.28. $f(z) = \dfrac{2z}{z^2+4},\ z_0 = 3+2i.$

13.29. $f(z) = \dfrac{2z}{z^2-4},\ z_0 = -1+3i.$

13.30. $f(z) = \dfrac{2z}{z^2-4},\ z_0 = 2+2i.$

13.31. $f(z) = \dfrac{2z}{z^2-4},\ z_0 = 3-2i.$

13.32. $f(z) = \dfrac{2z}{z^2-4},\ z_0 = 2+3i.$

13.33. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 1+2i.$

13.34. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 1+3i.$

13.35. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 1-3i.$

13.36. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 1-2i.$

13.37. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 2+i.$

13.38. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = 2-i.$

13.39. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = -2+i.$

13.40. $f(z) = \dfrac{3z}{z^2-9},\ z_0 = -1+3i.$

Задача 14. Вычислить интеграл.

14.1. $\displaystyle\oint\limits_{|z|=0.1} \frac{\cos z^3}{z^{14}}dz.$

14.2. $\displaystyle\oint\limits_{|z|=0.1} \frac{\sin z^4}{z^{14}}dz.$

14.3. $\displaystyle\oint\limits_{|z|=0.1} \frac{e^{z^4}}{z^{14}}dz.$

14.4. $\displaystyle\oint\limits_{|z|=0.1} \frac{\cos 3z^3}{z^{13}}dz.$

14.21. $\displaystyle\oint\limits_{|z|=0.1} \frac{e^{z^6}}{z^{15}}dz.$

14.22. $\displaystyle\oint\limits_{|z|=0.1} \frac{\cos 7z^5}{z^{16}}dz.$

14.23. $\displaystyle\oint\limits_{|z|=0.1} \frac{\sin 7z^5}{z^{16}}dz.$

14.24. $\displaystyle\oint\limits_{|z|=0.1} \frac{e^{7z^5}}{z^{16}}dz.$

14.5. $\oint\limits_{|z|=0.1} \frac{\sin 3z^4}{z^{13}}dz.$

14.6. $\oint\limits_{|z|=0.1} \frac{e^{3z^4}}{z^{13}}dz.$

14.7. $\oint\limits_{|z|=0.1} \frac{\cos 2z^4}{z^{15}}dz.$

14.8. $\oint\limits_{|z|=0.1} \frac{\sin 2z^4}{z^{15}}dz.$

14.9. $\oint\limits_{|z|=0.1} \frac{e^{2z^4}}{z^{15}}dz.$

14.10. $\oint\limits_{|z|=0.1} \frac{\cos 4z^5}{z^{13}}dz.$

14.11. $\oint\limits_{|z|=0.1} \frac{\sin 4z^3}{z^{13}}dz.$

14.12. $\oint\limits_{|z|=0.1} \frac{e^{4z^3}}{z^{13}}dz.$

14.13. $\oint\limits_{|z|=0.1} \frac{\cos 6z^4}{z^{14}}dz.$

14.14. $\oint\limits_{|z|=0.1} \frac{\sin 6z^4}{z^{14}}dz.$

14.15. $\oint\limits_{|z|=0.1} \frac{e^{6z^4}}{z^{14}}dz.$

14.16. $\oint\limits_{|z|=0.1} \frac{\cos 7z^6}{z^{17}}dz.$

14.17. $\oint\limits_{|z|=0.1} \frac{\sin 7z^3}{z^{17}}dz.$

14.18. $\oint\limits_{|z|=0.1} \frac{e^{7z^7}}{z^{17}}dz.$

14.25. $\oint\limits_{|z|=0.1} \frac{\cos z^4}{z^{15}}dz.$

14.26. $\oint\limits_{|z|=0.1} \frac{\sin z^4}{z^{15}}dz.$

14.27. $\oint\limits_{|z|=0.1} \frac{e^{z^3}}{z^{15}}dz.$

14.28. $\oint\limits_{|z|=0.1} \frac{\cos 2z^4}{z^{14}}dz.$

14.29. $\oint\limits_{|z|=0.1} \frac{\sin 2z^3}{z^{14}}dz.$

14.30. $\oint\limits_{|z|=0.1} \frac{e^{2z^3}}{z^{14}}dz.$

14.31. $\oint\limits_{|z|=0.1} \frac{\cos 3z^5}{z^{14}}dz.$

14.32. $\oint\limits_{|z|=0.1} \frac{\sin 3z^4}{z^{15}}dz.$

14.33. $\oint\limits_{|z|=0.1} \frac{e^{3z^4}}{z^{15}}dz.$

14.34. $\oint\limits_{|z|=0.1} \frac{\cos 4z^3}{z^{14}}dz.$

14.35. $\oint\limits_{|z|=0.1} \frac{\sin 4z^3}{z^{14}}dz.$

14.36. $\oint\limits_{|z|=0.1} \frac{e^{4z^5}}{z^{14}}dz.$

14.37. $\oint\limits_{|z|=0.1} \frac{e^{5z^4}}{z^{14}}dz.$

14.38. $\oint\limits_{|z|=0.1} \frac{\cos 5z^5}{z^{14}}dz.$

14.19. $\oint\limits_{|z|=0.1} \frac{\cos 5z^6}{z^{15}}dz.$

14.20. $\oint\limits_{|z|=0.1} \frac{\sin 5z^5}{z^{15}}dz.$

14.39. $\oint\limits_{|z|=0.1} \frac{\sin 5z^5}{z^{14}}dz.$

14.40. $\oint\limits_{|z|=0.1} \frac{e^{7z^4}}{z^{18}}dz.$

Задача 15. Вычислить интеграл.

15.1. $\oint\limits_{|z|=3} \left(z\cos\frac{1}{z} + \frac{\sin\frac{\pi z}{2}}{(z-2)^2(z-4i)} \right) dz.$

15.2. $\oint\limits_{|z|=3} \left(z\sin\frac{1}{z} + \frac{\cos\frac{\pi z}{4}}{(z-i)^2(z+5)} \right) dz.$

15.3. $\oint\limits_{|z+3|=4} \left(ze^{\frac{1}{z}} + \frac{\mathrm{sh}z}{(z+2)^2(z-3i)} \right) dz.$

15.4. $\oint\limits_{|z|=1} \left(z^2\cos\frac{2}{z} + \frac{\sin 2\pi z}{(z-0.5i)^2(z-1)} \right) dz.$

15.5. $\oint\limits_{|z|=1} \left(z^2\sin\frac{2}{z} + \frac{\cos\pi z}{(z+0.5)^2(z-2i)} \right) dz.$

15.6. $\oint\limits_{|z|=4} \left(z^2e^{\frac{2}{z}} + \frac{\mathrm{ch}z}{(z-2)(z-3i)^2(z-5)} \right) dz.$

15.7. $\oint\limits_{|z|=3} \left(z^2\cos\frac{1}{z} + \frac{\sin\pi z}{(z-1)^2(z-2i)(z-6)} \right) dz.$

15.8. $\oint\limits_{|z|=1} \left(z^2\sin\frac{2}{z} + \frac{\cos z}{(z-0.6i)^2(z-0.5)(z-5)} \right) dz.$

15.9. $\oint\limits_{|z|=4} \left(z^5e^{\frac{3}{z}} + \frac{\mathrm{sh}z}{(z-3)^2(z-2i)(z-5)} \right) dz.$

15.10. $\oint\limits_{|z|=1} \left(z^3\cos\frac{3}{z} + \frac{\sin\pi z}{(z-2i)(z-0.5)^3} \right) dz.$

15.11. $\oint\limits_{|z|=2} \left(z^5\sin\frac{4}{z} + \frac{\cos\frac{\pi z}{2}}{(z-i)^3(z-3)} \right) dz.$

15.12. $\oint\limits_{|z|=1} \left(z^4 e^{\frac{5}{z}} + \dfrac{\operatorname{sh} z}{(z-0.3i)^2(z-3)} \right) dz.$

15.13. $\oint\limits_{|z|=2} \left(z^4 \cos\dfrac{6}{z} + \dfrac{\sin \pi z}{(z-1)^3(z-3i)} \right) dz.$

15.14. $\oint\limits_{|z|=5} \left(z^4 \sin\dfrac{6}{z} + \dfrac{\cos\dfrac{\pi z}{10}}{(z-2i)^3(z-4)} \right) dz.$

15.15. $\oint\limits_{|z|=3} \left(z^4 e^{\frac{6}{z}} + \dfrac{\operatorname{ch} 3z}{(z-2)(z-i)^2(z-4)} \right) dz.$

15.16. $\oint\limits_{|z|=2} \left(z^4 \cos\dfrac{2}{z} + \dfrac{\sin 2\pi z}{(z-3i)^2(z-1)} \right) dz.$

15.17. $\oint\limits_{|z|=1} \left(z^5 \sin\dfrac{5}{z} + \dfrac{\cos\dfrac{\pi z}{2}}{(z-0.5i)^2(z-4)} \right) dz.$

15.18. $\oint\limits_{|z|=2} \left(z^4 e^{\frac{4}{z}} + \dfrac{\sin \pi i z}{(z-i)^3(z-4)} \right) dz.$

15.19. $\oint\limits_{|z|=3} \left(z^7 \cos\dfrac{4}{z} + \dfrac{\sin 2z}{(z-2i)^3(z-5)} \right) dz.$

15.20. $\oint\limits_{|z|=1} \left(z^4 \sin\dfrac{1}{z} + \dfrac{\cos z}{(z-3)(z-0.8i)^3} \right) dz.$

15.21. $\oint\limits_{|z|=1} \left(z^4 e^{\frac{2}{z}} + \dfrac{\operatorname{ch} iz}{(z-2i)(z-0.6)^2} \right) dz.$

15.22. $\oint\limits_{|z|=2} \left(z^4 \cos\dfrac{1}{z} + \dfrac{\sin \pi z}{(z-1)^3(z-4i)} \right) dz.$

15.23. $\oint\limits_{|z|=2} \left(z^4 \sin\dfrac{1}{z} + \dfrac{\sin\dfrac{\pi z}{2}}{(z-1)^3(z-4i)} \right) dz.$

15.24. $\oint\limits_{|z|=3} \left(z^2 e^{\frac{1}{z}} + \dfrac{\operatorname{sh} \pi i z}{(z-i)^3(z-4)} \right) dz.$

15.25. $\oint\limits_{|z|=3} \left(z^2 \cos \frac{3}{z} + \frac{\mathrm{sh}\pi iz}{(z-1)^3(z-4i)} \right) dz.$

15.26. $\oint\limits_{|z|=1} \left(z^2 \sin \frac{1}{z} + \frac{\mathrm{ch}\pi iz}{(z-1)^3(z-4i)} \right) dz.$

15.27. $\oint\limits_{|z|=1} \left(z^3 e^{\frac{3}{z}} + \frac{\mathrm{ch}\pi iz}{(z-0.4)^3(z-2i)} \right) dz.$

15.28. $\oint\limits_{|z|=2} \left(z^3 \cos \frac{2}{z} + \frac{\mathrm{sh}iz}{(z-1)^3(z-3i)} \right) dz.$

15.29. $\oint\limits_{|z|=2} \left(z^5 \sin \frac{2}{z} + \frac{\mathrm{ch}\dfrac{\pi iz}{2}}{(z-0.5)^3(z-3i)} \right) dz.$

15.30. $\oint\limits_{|z|=3} \left(z^4 e^{\frac{3}{z}} + \frac{\sin \frac{\pi iz}{2}}{(z-2i)^3(z-4)} \right) dz.$

15.31. $\oint\limits_{|z|=6} \left(z^5 \cos \frac{4}{z} + \frac{\mathrm{sh}\pi iz}{(z-2i)^2(z-7)} \right) dz.$

15.32. $\oint\limits_{|z|=3} \left(z^4 \sin \frac{5}{z} + \frac{\mathrm{ch}\pi iz}{(z-i)^2(z-1)^2} \right) dz.$

15.33. $\oint\limits_{|z|=3} \left(z^2 e^{\frac{3}{z}} + \frac{\sin iz}{(z-i)^2(z-2)} \right) dz.$

15.34. $\oint\limits_{|z|=3} \left(z^4 \cos \frac{3}{z} + \frac{\mathrm{sh}\dfrac{\pi iz}{3}}{(z-2i)^2(z-4)} \right) dz.$

15.35. $\oint\limits_{|z|=2} \left(z^3 \sin \frac{2}{z} + \frac{\mathrm{ch}\pi iz}{(z-i)^3(z-3)} \right) dz.$

15.36. $\oint\limits_{|z|=2} \left(z^5 e^{\frac{7}{z}} + \frac{\cos iz}{z^3(z-3i)} \right) dz.$

15.37. $\oint\limits_{|z|=1} \left(z^5 \cos \frac{1}{z} + \frac{\mathrm{sh}\dfrac{\pi iz}{2}}{(z-0.5)^3(z-2i)} \right) dz.$

15.38. $\oint\limits_{|z|=2} \left(z^5 \sin\frac{1}{z} + \frac{\mathrm{ch}\frac{\pi i z}{2}}{(z-1)^4(z-5i)} \right) dz.$

15.39. $\oint\limits_{|z|=3} \left(z^5 e^{\frac{1}{z}} + \frac{\mathrm{ch}\pi i z}{(z-2i)^2(z-4)} \right) dz.$

15.40. $\oint\limits_{|z|=3} \left(z^3 \cos 5z + \frac{\mathrm{sh} i z}{(z-1)^3(z-3i)} \right) dz.$

Задача 16. Вычислить интеграл.

16.1. $\int\limits_{|z+1|=2} \frac{1}{(z^2+1)^2(z^2-4)} dz.$

16.2. $\int\limits_{|z-2|=1.5} \frac{z^5}{(z^2-4z+5)^2(z^2-1)} dz.$

16.3. $\int\limits_{|z|=1.5} \frac{(z^6-z^5)e^{2/z}}{z^7-1} dz.$

16.4. $\int\limits_{|z+2|=2} \frac{z^5-z^2}{(z^2+2z+2)^2(z^2-4)} dz.$

16.5. $\int\limits_{|z-1|=2} \frac{1}{(z^2+z-2)(2z^2+1)^2} dz.$

16.6. $\int\limits_{|z|=3} \frac{z^4\cos(2/z)}{z^5+1} dz.$

16.7. $\int\limits_{|z|=2} \frac{z^{14}}{(z^6-1)^2(z^2+2z+2)} dz.$

16.8. $\int\limits_{|z|=1.5} \frac{z^5\sin(1/z^2)}{z^4+3z^2+1} dz.$

16.9. $\int\limits_{|z|=2} \frac{1}{(z^8+2)(z^2+1)} dz.$

16.10. $\int\limits_{|z-1|=3} \frac{1}{(z^2-3)^2(z^2+4z+3)} dz.$

16.11. $\int\limits_{|z|=2} \frac{z^{17}}{(z^2+1)^3(z^3+2)^4}dz.$

16.12. $\int\limits_{|z|=1.5} \frac{dz}{(z^4-1)(z+2)}$

16.13. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^4-1)(z-2)}.$

16.14. $\int\limits_{|z|=2} \frac{z^{15}+z^3}{z^4+1}dz.$

16.15. $\int\limits_{|z|=3} \frac{z^8}{z^9-1}dz.$

16.16. $\int\limits_{|z|=3} \frac{z^{15}}{(z^2+2)^2(z^3+3)^4}dz.$

16.17. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^2-1)(z^2-2z+2)}.$

16.18. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^3+1)(z^2-2z+2)}.$

16.19. $\int\limits_{|z|=2} \frac{z^6+z^4}{z^5+1}dz.$

16.20. $\int\limits_{|z|=2} \frac{z^7}{z^8-1}dz.$

16.21. $\int\limits_{|z|=3} \frac{z^{15}}{(z^2-2)^2(z^3+3)^4}dz.$

16.22. $\int\limits_{|z+1|=1.5} \frac{dz}{(z^3-1)(z+2)}.$

16.23. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^3+1)z(z-2)}.$

16.24. $\int\limits_{|z|=2} \frac{z^7+z^5}{z^6+1}dz.$

16.25. $\int\limits_{|z|=3} \frac{z^3 e^{1/z}}{(z^2+4)^2} dz.$

16.26. $\int\limits_{|z|=2} \frac{z^{17}}{(z^3+2)(z^4-1)^3} dz.$

16.27. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^3-1)(z^2-2z+2)}.$

16.28. $\int\limits_{|z+1|=1.5} \frac{dz}{(z^4-1)(z^2+2z+2)}.$

16.29. $\int\limits_{|z|=2} \frac{z^8+z^6}{z^7+1} dz.$

16.30. $\int\limits_{|z|=2} \frac{z^2 e^{1/z}}{(z^2+1)^2} dz.$

16.31. $\int\limits_{|z|=3} \frac{\sin(1/z)}{z(z+1)^2(z+2)(z+4)} dz.$

16.32. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^4-1)^2(z^2-2z+2)}.$

16.33. $\int\limits_{|z-1|=1.5} \frac{dz}{(z^4-1)(z^2+2z+2)(z-2)}.$

16.34. $\int\limits_{|z|=2} \frac{z^9+z^7}{z^8+1} dz.$

16.35. $\int\limits_{|z|=2} \frac{dz}{1+z^{10}}.$

16.36. $\int\limits_{|z|=1} \frac{z^{10}+1}{z^{11}} dz.$

16.37. $\int\limits_{|z|=2} \frac{z^9}{z^{10}-1} dz.$

16.38. $\int\limits_{|z|=2} \frac{dz}{1+z^{12}}.$

16.39. $\displaystyle\int\limits_{|z|=3} \frac{z^8}{(z^2+2)^3(z-2)^3}dz.$

16.40. $\displaystyle\int\limits_{|z|=2} \frac{\sin(1/z)}{z^4-1}dz.$

Задача 17. Вычислить интеграл.

17.1. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{2+\sin x}.$

17.2. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{3+2\sin x}.$

17.3. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{3+\sin x}.$

17.4. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{4+3\sin x}.$

17.5. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{4+\sin x}.$

17.6. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{5+4\sin x}.$

17.7. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{5+\sin x}.$

17.8. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{6+5\sin x}.$

17.9. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{6+\sin x}.$

17.10. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{7+6\sin x}.$

17.21. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{12+\sin x}.$

17.22. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{13+12\sin x}.$

17.23. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{13+\sin x}.$

17.24. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{14+13\sin x}.$

17.25. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{14+\sin x}.$

17.26. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{15+14\sin x}.$

17.27. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{15+\sin x}.$

17.28. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{16+15\sin x}.$

17.29. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{16+\sin x}.$

17.30. $\displaystyle\int\limits_0^{2\pi} \frac{dx}{17+16\sin x}.$

17.11. $\int\limits_0^{2\pi} \frac{dx}{7+\sin x}$.

17.12. $\int\limits_0^{2\pi} \frac{dx}{8+7\sin x}$.

17.13. $\int\limits_0^{2\pi} \frac{dx}{8+\sin x}$.

17.14. $\int\limits_0^{2\pi} \frac{dx}{9+8\sin x}$.

17.15. $\int\limits_0^{2\pi} \frac{dx}{9+\sin x}$.

17.16. $\int\limits_0^{2\pi} \frac{dx}{10+9\sin x}$.

17.17. $\int\limits_0^{2\pi} \frac{dx}{10+\sin x}$.

17.18. $\int\limits_0^{2\pi} \frac{dx}{11+10\sin x}$.

17.19. $\int\limits_0^{2\pi} \frac{dx}{11+\sin x}$.

17.20. $\int\limits_0^{2\pi} \frac{dx}{12+11\sin x}$.

17.31. $\int\limits_0^{2\pi} \frac{dx}{17+\sin x}$.

17.32. $\int\limits_0^{2\pi} \frac{dx}{18+17\sin x}$.

17.33. $\int\limits_0^{2\pi} \frac{dx}{18+\sin x}$.

17.34. $\int\limits_0^{2\pi} \frac{dx}{19+18\sin x}$.

17.35. $\int\limits_0^{2\pi} \frac{dx}{19+\sin x}$.

17.36. $\int\limits_0^{2\pi} \frac{dx}{20+19\sin x}$.

17.37. $\int\limits_0^{2\pi} \frac{dx}{20+\sin x}$.

17.38. $\int\limits_0^{2\pi} \frac{dx}{21+20\sin x}$.

17.39. $\int\limits_0^{2\pi} \frac{dx}{21+\sin x}$.

17.40. $\int\limits_0^{2\pi} \frac{dx}{22+21\sin x}$.

Задача 18. Вычислить интеграл.

18.1. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+81)}$.

18.2. $\int\limits_{-\infty}^{\infty} \frac{\cos 2x dx}{(x^2+1)^2(x^2+81)}$.

18.21. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+49)^2(x^2+16)}$.

18.22. $\int\limits_{-\infty}^{\infty} \frac{\sin 4x dx}{(x^2+6x+58)^2}$.

18.3. $\int\limits_{-\infty}^{\infty} \frac{\sin 3x dx}{(x^2+2x+17)^2}$.

18.4. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+4)}$.

18.5. $\int\limits_{-\infty}^{\infty} \frac{\sin x dx}{(x^2+4x+20)^2}$.

18.6. $\int\limits_{-\infty}^{\infty} \frac{\cos 4x dx}{(x^2+1)^2(x^2+16)}$.

18.7. $\int\limits_{-\infty}^{\infty} \frac{\sin 5x dx}{(x^2+6x+25)^2}$.

18.8. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+36)}$.

18.9. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^+16)^2(x^2+121)}$.

18.10. $\int\limits_{-\infty}^{\infty} \frac{\sin 5x dx}{(x^2+4x+29)^2}$.

18.11. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+25)}$.

18.12. $\int\limits_{-\infty}^{\infty} \frac{\cos 4x dx}{(x^2+25)^2(x^2+121)}$.

18.13. $\int\limits_{-\infty}^{\infty} \frac{\sin 2x dx}{(x^2+8x+41)^2}$.

18.14. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+16)^2(x^2+36)}$.

18.15. $\int\limits_{-\infty}^{\infty} \frac{\cos 5x dx}{(x^2+9)^2(x^2+1)}$.

18.23. $\int\limits_{-\infty}^{\infty} \frac{\cos 5x dx}{(x^2+81)^2(x^2+16)}$.

18.24. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+64)^2(x^2+9)}$.

18.25. $\int\limits_{-\infty}^{\infty} \frac{\cos 15x dx}{(x^2+1)^2(x^2+9)}$.

18.26. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+1)^2(x^2+121)}$.

18.27. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+25)^2(x^2+81)}$.

18.28. $\int\limits_{-\infty}^{\infty} \frac{\sin 3x dx}{(x^2+6x+25)^2}$.

18.29. $\int\limits_{-\infty}^{\infty} \frac{\cos 4x dx}{(x^2+4)^2(x^2+121)}$.

18.30. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+4)^2(x^2+144)}$.

18.31. $\int\limits_{-\infty}^{\infty} \frac{\sin x dx}{(x^2+8x+41)^2}$.

18.32. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+9)^2(x^2+100)}$.

18.33. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+25)^2(x^2+36)}$.

18.34. $\int\limits_{-\infty}^{\infty} \frac{\sin 3x dx}{(x^2+4x+11)^2}$.

18.35. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+16)^2(x^2+81)}$.

18.16. $\int\limits_{-\infty}^{\infty} \frac{\sin 4x dx}{(x^4+14x+74)^2}$.

18.17. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+121)}$.

18.18. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+4)^2(x^2+121)}$.

18.19. $\int\limits_{-\infty}^{\infty} \frac{\cos 12x dx}{(x^2+81)^2(x^2+144)}$.

18.20. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2(x^2+169)}$.

18.36. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+16)^2(x^2+25)}$.

18.37. $\int\limits_{-\infty}^{\infty} \frac{\sin 2x dx}{(x^2+4x+18)^2}$.

18.38. $\int\limits_{-\infty}^{\infty} \frac{\cos 5x dx}{(x^2+16)^2(x^2+81)}$.

18.39. $\int\limits_{-\infty}^{\infty} \frac{dx}{(x^2+36)^2(x^2+16)}$.

18.40. $\int\limits_{-\infty}^{\infty} \frac{\cos 3x dx}{(x^2+9)^2(x^2+196)}$.

Задача 19. Найти образ области D при отображении $w(z)$.

19.1. $D = \{\mathrm{Re}\ z > 1\},\ w = (2+2i)z + 1.$

19.2. $D = \{|z| < 1/2,\ -\pi/4 < z < \pi/3,\},\ w = 1/z + 1.$

19.3. $D = \{\mathrm{Re}\ z < 1\},\ w = (2\sqrt{3}i - 2)z - 3.$

19.4. $D = \{\mathrm{Re}\ z - \mathrm{Im}\ z > 0\},\ w = 2 + 1/z.$

19.5. $D = \{|z| < 1, \pi/4 < \arg z < 3\pi/4\},\ w = 2 - (\sqrt{2} + \sqrt{2}i)z.$

19.6. $D = \{\mathrm{Im}\ z - \mathrm{Re}\ z < 3\},\ w = (-3i - 3)z + 1.$

19.7. $D = \{1 < |z| < 2,\ \pi/3 < \arg z < \pi\},\ w = (\sqrt{3} + i)z + i.$

19.8. $D = \{\mathrm{Im}\ z < -1\},\ w = (1 - i)z + 2i.$

19.9. $D = \{1/3 < |z| < 1/2\},\ \pi/4 \leqslant \arg z \leqslant 3\pi/4,\ w = 2i + 1/z$

19.10. $D = \{\mathrm{Re}\ z < -1\},\ w = 1 + (3 - 3\sqrt{3}i)z.$

19.11. $D = \{\mathrm{Re}\ z + \mathrm{Im}\ z < 0\},\ w = i + 1/z.$

19.12. $D = \{|z| > 4,\ \pi/4 < \arg z < \pi/2\},\ w = -1 + i + \frac{1+i}{2\sqrt{2}}z.$

19.13. $D = \{|z| < 2,\ \mathrm{Re}\ z - \mathrm{Im}\ z > 0\},\ w = 2 + 2/z.$

19.14. $D = \{\text{Re } z > \text{Im } z\},\ w = 2iz + 3.$

19.15. $D = \{|z| < 1/2,\ \text{Im } z > 0\},\ w = 2i + 1/z.$

19.16. $D = \{\text{Re } z < -2\},\ w = (2i - 2)z + 1 + i$

19.17. $D = \{\text{Im } z - 2\text{Re } z > 0\},\ w = 1 + i + 1/z.$

19.18. $D = \{\text{Im } z < -2\},\ w = (2 + 2\sqrt{3}i)z - 1.$

19.19. $D = \{|z| > 1/2,\ \pi/3 < \arg z > 2\pi/3\},\ w = 1 + i + 1/z.$

19.20. $D = \{\text{Re } z + \text{Im } z < 0\},\ w = 1 - 3iz.$

19.21. $D = \{\text{Re } z < 1\},\ w = (2i - 2\sqrt{3})z + 1 + i.$

19.22. $D = \{2 < |z| < 4,\ \text{Re}z > 0,\ \text{Im}z > 0\},\ w = 1 + i - iz/2.$

19.23. $D = \{\text{Re}z + \text{Im } z > 1\},\ w = (3 + 3i)z + i.$

19.24. $D = \{\text{Im } z > 2\},\ w = 3i - (1 + i)z.$

19.25. $D = \{1/2 < |z| < 1,\ \text{Re}z > 0,\ \text{Im } z > 0\},\ w = 2 + 1/z.$

19.26. $D = \{\text{Im } z < -1\},\ w = 2 - (\sqrt{3} + i)z.$

19.27. $D = \{\text{Re } z + \text{Im } z < 1\},\ w = 3iz + 1 + i.$

19.28. $D = \{\text{Re } z > 1,\ w = (3\sqrt{3} - 3i)z + 1\}.$

19.29. $D = \{\text{Im } z + 2\text{Re } z < 0\},\ w = -1 + 1/z.$

19.30. $D = \{\text{Im } z < -3\},\ w = i - (1 + \sqrt{3}i)z.$

19.31. $D = \{\text{Re } z - \text{Im } z < 1\},\ w = (1 + i)z + 2 + i.$

19.32. $D = \{|z| < 2,\ \pi/2 < \arg z < 2\pi/3\},\ w = 1 + \dfrac{1 + i\sqrt{3}}{4}z.$

19.33. $D = \{|z| > 1/3,\ \pi/2 < \arg z < 3\pi/4\},\ w = 3i + 1/z.$

19.34. $D = \{\text{Re } z < 3\},\ w = (\sqrt{3} + i)z + 2.$

19.35. $D = \{\text{Re } z - \text{Im } z < -1\},\ w = 2 - 4iz.$

19.36. $D = \{\text{Re } z > 1\},\ w = (1 + \sqrt{3}i)z + 2i.$

19.37. $D = \{2\text{Re } z - \text{Im } z > 0\},\ w = 5iz + 2.$

19.38. $D = \{\mathrm{Im}\ z > -1\},\ w = (2\sqrt{3} - 2i)z - 1.$

19.39. $D = \{|z| < 1,\ \mathrm{Re}\ z + \mathrm{Im}\ z < 0\},\ w = 1 + 2/z.$

19.40. $D = \{\mathrm{Re}\ z + \mathrm{Im}\ z > 2\},\ w = (i-1)z + 2i.$

Задача 20. Найти дробно-линейную функцию $w(z)$, конформно отображающую область D на область G и удовлетворяющую указанным условиям.

20.1. $D = \{|z| > 1\},\ G = \{|w| < 2\},\ w(i) = -2,\ w(-2i) = 0.$

20.2. $D = \{\mathrm{Im}\ z > 1\},\ G = \{|w| < 3\},\ w(i) = 3,\ w(2i) = 0.$

20.3. $D = \{\mathrm{Re}\ z < 2\},\ G = \{|w| > 1\},\ w(2) = -1,\ w(0) = \infty.$

20.4. $D = \{|z| < 1\},\ G = \{\mathrm{Im}\ w < 1\},\ w(0) = -2i,\ w(1) = i.$

20.5. $D = \{|z| < 1\},\ G = \{\mathrm{Re}\ w > 1\},\ w(0) = 2,\ w(1) = 1.$

20.6. $D = \{|z| < 2\},\ G = \{|w| > 1\},\ w(2) = -1,\ w(i) = -2.$

20.7. $D = \{\mathrm{Im}\ z > 1\},\ G = \{\mathrm{Re}\ w - \mathrm{Im}\ w > -1\},\ w(1) = i,\ w(2i) = 0.$

20.8. $D = \{\mathrm{Re}\ z > 2\},\ G = \{|w| < 2\},\ w(2) = 2i,\ w(4) = 0.$

20.9. $D = \{|z| < 3\},\ G = \{\mathrm{Im}\ w > 2\},\ w(0) = 4i,\ w(3i) = 2i.$

20.10. $D = \{\mathrm{Re}\ z < -1\},\ G = \{|w| < 3\},\ w(-1) = 3i,\ w(-2) = 0.$

20.11. $D = \{|z| < 1\},\ G = \{|w| < 2\},\ w(-1) = -2i,\ w(0) = 1.$

20.12. $D = \{\mathrm{Im}\ z < -2\},\ G = \{|w| < 2\},\ w(-2i) = 2,\ w(-4i) = 0.$

20.13. $D = \{|z| < 3\},\ G = \{\mathrm{Re}\ w > 4\},\ w(0) = 8,\ w(3) = 4.$

20.14. $D = \{|z| < 2\},\ G = \{\mathrm{Im}\ w > 2\},\ w(0) = 4i,\ w(2) = 2i.$

20.15. $D = \{\mathrm{Im}\ z < -2\},\ G = \{\mathrm{Im}\ w + 2 < \mathrm{Re}\ w\},$

$w(-2) = -2i,\ w(-4i) = -2 - 2i.$

20.16. $D = \{\mathrm{Im}\ z > 4\},\ G = \{|w| < 2\},\ w(4i) = 2,\ w(8i) = 0.$

20.17. $D = \{|z| > 1\},\ G = \{\mathrm{Re}\ w < -1\},\ w(1) = -1,\ w(\infty) = -2.$

20.18. $D = \{\text{Im } z > 2\},\ G = \{\text{Im } w < -3\},\ w(\infty) = -6i,\ w(2) = -3i.$

20.19. $D = \{|z| > 2\},\ G = \{|w| < 4\},\ w(2i) = 4,\ w(4) = i.$

20.20. $D = \{\text{Re } z > 3\},\ G = \{|w| < 1\},\ w(3) = i,\ w(6) = 0.$

20.21. $D = \{\text{Im } z < 1\},\ G = \{|w| < 1\},\ w(i) = 1,\ w(0) = 0.$

20.22. $D = \{|z| > 1\},\ G = \{|w| > 3\},\ w(-1) = 3i,\ w(-2i) = \infty.$

20.23. $D = \{|z| > 2\},\ G = \{\text{Im } w < 3\},\ w(2) = 3i,\ w(\infty) = 0.$

20.24. $D = \{\text{Re } z > -1\},\ G = \{\text{Re } w + 1 > \text{Im } w\},$

$w(1) = 0,\ w(-1 + i) = -i.$

20.25. $D = \{|z| < 2\},\ G = \{\text{Im } w < -3\},\ w(0) = -6i,\ w(2) = -3i.$

20.26. $D = \{|z| > 3\},\ G = \{\text{Im } w > 2\},\ w(\infty) = 4i,\ w(3i) = 2i.$

20.27. $D = \{\text{Im } z < 2\},\ G = \{|w| > 1\},\ w(2i) = 1,\ w(0) = \infty.$

20.28. $D = \{\text{Re } z < 3\},\ G = \{|w| > 2\},\ w(3) = 2i,\ w(0) = \infty.$

20.29. $D = \{|z| < 3\},\ G = \{|w| < 2\},\ w(3i) = -2,\ w(0) = i.$

20.30. $D = \{\text{Re } z < 2\},\ G = \{\text{Re } w > \text{Im } w + 2\},$

$w(2 + i) = 2,\ w(1) = 2 - 2i.$

20.31. $D = \{|z| < 3\},\ G = \{\text{Im } w < -3\},\ w(0) = -6i,\ w(3) = -3i.$

20.32. $D = \{|z| < 4\},\ G = \{\text{Re } w > 3\},\ w(0) = 6,\ w(4) = 3.$

20.33. $D = \{\text{Re } z < 2\},\ G = \{|w| > 4\},\ w(2) = 4i,\ w(0) = \infty.$

20.34. $D = \{|z| > 1\},\ G = \{\text{Re } z > 1\},\ w(i) = 1,\ w(-2) = 2.$

20.35. $D = \{|z| > 2\},\ G = \{|w| > 1\},\ w(-2i) = 4,\ w(\infty) = 2i.$

20.36. $D = \{\text{Re } z > 4\},\ G = \{|w| < 2\},\ w(4) = 2i,\ w(8) = 0.$

20.37. $D = \{\text{Im } z > 2\},\ G = \{|w| > 2\},\ w(1 + 2i) = 2,\ w(4) = \infty.$

20.38. $D = \{\text{Re } z > \text{Im } z\},\ G = \{|w| < 1\},\ w(-1 + i) = 0,\ w(0) = 1.$

20.39. $D = \{|z| < 1\},\ G = \{\mathrm{Im}\ z > 4\},\ w(0) = 8i,\ w(1) = 2 + 4i.$

20.40. $D = \{|z| < 4\},\ G = \{\mathrm{Re}\ w > 1\},\ w(4i) = 1 + i,\ w(0) = 2.$

Задача 21. Найти образ области D при конформном отображении.

21.1. $D = \{\alpha < \mathrm{Im}\ z < \beta,\ 0 < \alpha < \beta < 2\pi\},\ w = e^z.$

21.2. $D = \{0 < \mathrm{Re}\ z < \pi,\ \mathrm{Im}\ z < 0\},\ w = \cos z.$

21.3. $D = \{0 < \arg z < \pi/4\},\ w = \ln z.$

21.4. $D = \{|z| < 1,\ \mathrm{Im}\ z > 0\}, w = \dfrac{1 - z}{1 + z}$

21.5. $D = \{\pi < \mathrm{Im}\ z < 2\pi\},\ w = \mathrm{sh}z.$

21.6. $D = \{0 < \mathrm{Im}\ z - \mathrm{Re}\ z < 2\pi\},\ w = e^z.$

21.7. $D = \{0 < \mathrm{Re}\ z < \pi/2,\ \mathrm{Im}\ z > 0\},\ w = \cos z.$

21.8. $D = \{|z| < 1,\ \pi/2 < \arg z < \pi\},\ w = \ln z.$

21.9. $D = \{\mathrm{Re}\ z < 1\},\ w = \dfrac{z - 3 + i}{z + 1 + i}.$

21.10. $D = \{\mathrm{Re}\ z > 0,\ \pi/2 < \mathrm{Im}z < \pi\},\ w = \mathrm{sh}z.$

21.11. $D = \{\mathrm{Re}\ z < 0,\ 0 < \mathrm{Im}\ z < \pi\},\ w = e^z.$

21.12. $D = \{-\pi/2 < \mathrm{Re}z < \pi/2,\ \mathrm{Im}\ z > 0\},\ w = \cos z.$

21.13. $D = \{1 < |z| < 2\} \setminus [1, 2],\ w = \ln z.$

21.14. $D = \{\mathrm{Re}\ z < 0,\ -\pi/2 < \mathrm{Im}z < \pi/2\},\ w = \mathrm{sh}z.$

21.15. $D = \{1 < |z| < 2\},\ w = \dfrac{2}{z - 1}.$

21.16. $D = \{\mathrm{Re}\ z > 0,\ 0 < \mathrm{Im}\ z < 3\pi/4\},\ w = e^z.$

21.17. $D = \{-\pi/4 < \mathrm{Re}\ z < \pi/4\},\ w = \mathrm{tg}z.$

21.18. $D = \{0 < \mathrm{Im}\ z < \pi\},\ w = \mathrm{ch}z.$

21.19. $D = \{\pi/2 < \mathrm{Re}\ z < \pi\ \ \mathrm{Im}\ z < 0\},\ w = \sin z.$

21.20. $D = \{-1 < \mathrm{Re}\ z + \mathrm{Im}\ z < 0\},\ w = \dfrac{i - z}{z}.$

21.21. $D = \{0 < \mathrm{Re}\ z < \pi\},\ w = \cos z.$

21.22. $D = \{0 < \mathrm{Re}\ z < \pi\},\ w = \mathrm{tg}z.$

21.23. $D = \{\mathrm{Re}\ z > 1/2,\ \pi < \mathrm{Im}z < 2\pi\},\ w = e^z.$

21.24. $D = \{0 < \mathrm{Re}\ z < 1,\ \mathrm{Im}\ z > 0\},\ w = \dfrac{z+1}{z}.$

21.25. $D = \{0 < \mathrm{Re}\ z < \pi,\ -1 < \mathrm{Im}\ z < 1\},\ w = \cos z.$

21.26. $D = \{\mathrm{Re}\ z > 0,\ \mathrm{Im}\ z > 0\},\ w = \mathrm{arcsin}z.$

21.27. $D = \{0 < \mathrm{Re}\ z < \pi,\ \mathrm{Im}\ z > 0\},\ w = \mathrm{tg}z.$

21.28. $D = \{\mathrm{Re}\ z > 0,\ 0 < \mathrm{Im}\ z < \pi\},\ w = \mathrm{ch}z.$

21.29. $D = \{\mathrm{Re}\ z < 0,\ -1 < \mathrm{Im}\ z < 0\},\ w = \dfrac{z-2}{z}.$

21.30. $D = \{\mathrm{Re}\ z > 0,\ 0 < \mathrm{Im}\ z < \pi\},\ w = \mathrm{cth}z.$

21.31. $D = \{\pi < \mathrm{Re}\ z < 2\pi,\ \mathrm{Im}\ z > 0\},\ w = \sin z.$

21.32. $D = \{0 < \mathrm{Re}\ z < 2,\ 0 < \mathrm{Im}\ z < \pi/2\},\ w = \mathrm{ch}z.$

21.33. $D = \{-1 < \mathrm{Im}\ z - \mathrm{Re}\ z < 0\},\ w = \dfrac{z-1}{z}.$

21.34. $D = \{\mathrm{Im}\ z > 0\},\ w = \mathrm{arcsin}z.$

21.35. $D = \{0 < \mathrm{Im}\ z - 2\mathrm{Re}\ z < 2\pi\},\ w = e^z.$

21.36. $D = \{0 < \mathrm{Re}\ z < \pi/2,\ 0 < \mathrm{Im}\ z < 1\},\ w = \sin z.$

21.37. $D = \{0 < \mathrm{Im}\ z < \pi\},\ w = \mathrm{cth}z.$

21.38. $D = \{\pi < \mathrm{Re}\ z < 2\pi,\ -1 < \mathrm{Im}\ z < 1\},\ w = \mathrm{sh}z.$

21.39. $D = \{|z - i| < 1,\ \mathrm{Im}\ z > 0\},\ w = \dfrac{z+i}{z}.$

21.40. $D = \{\mathrm{Im}\ z < 0\},\ w = \ln z.$

Задача 22. Найти изображение $F(p)$ следующих оригиналов.

22.1. $t^2 \sin t.$

22.2. $e^{t-2}\cos(t-2)\eta(t-2).$

22.21. $t^2\mathrm{sh}t.$

22.22. $(t-2)\mathrm{sh}(3t-6)\eta(t-2).$

22.3. $t^2 \cos t$.

22.4. $\mathrm{ch}(t-1)\cos(t-1)\eta(t-1)$.

22.5. $t\mathrm{sh}3t$.

22.6. $(t-1)\mathrm{sh}(3t-3)\eta(t-1)$.

22.7. $t\mathrm{sh}5t$.

22.8. $\mathrm{sh}(t-1)\cos(t-1)\eta(t-1)$.

22.9. $e^{-2t}\cos^2 t$.

22.10. $\mathrm{sh}(t-1)\cos(t-1)\eta(t-1)$.

22.11. $t\sin^2 t$.

22.12. $t\mathrm{ch}5t$.

22.13. $e^{t-1}\eta(t-1)$.

22.14. $e^{3t}\sin^2 t$.

22.15. $\mathrm{ch}(t-1)\sin(t-1)\eta(t-1)$.

22.16. $\mathrm{ch}t\cos t$.

22.17. $\mathrm{ch}t\sin t$.

22.18. $(t-1)\cos(t-1)\eta(t-1)$.

22.19. $t\cos^2 t$.

22.20. $e^{8t-8}(t-1)\eta(t-1)$.

22.23. $te^t\cos t$.

22.24. $t\mathrm{ch}3t$.

22.25. $\mathrm{sh}t\sin t$.

22.26. t^2e^{8t}.

22.27. $te^{-t}\sin t$.

22.28. $e^t\mathrm{ch}t$.

22.29. $t^2\mathrm{ch}t$.

22.30. $\mathrm{sh}t\cos t$.

22.31. $e^{2t}\cos 2t$.

22.32. $e^{2t}\sin 2t$.

22.33. $\mathrm{sh}3t\sin 2t$.

22.34. $e^{t-3}\eta(t-3)$.

22.35. $t^2\cos^2 t$.

22.36. $e^{-2t}\sin 2t$.

22.37. $t^2\mathrm{ch}5t$.

22.38. $e^{4t}\cos^2 t$

22.39. $t\mathrm{sh}4t$.

22.40. t^2e^{5t}.

Задача 23. Найти оригиналы по изображению.

23.1. $\dfrac{e^{-p}}{p^2-1}$.

23.2. $\dfrac{p^2-7p+14}{(p^2-6p-13)(p-1)}$.

23.21. $\dfrac{e^{-2p}}{p^2-4p+3}$.

23.22. $\dfrac{2}{(p^2-6p-13)(p-3)}$.

23.3. $\dfrac{e^{-p}}{(p-1)(p-6)}$.

23.4. $\dfrac{3p-15}{(p^2-4p+13)(p+1)}$.

23.5. $\dfrac{e^{-p}}{p(p^2-1)}$.

23.6. $\dfrac{8p+14}{(p-2)(p-3)(p+1)}$.

23.7. $\dfrac{e^{-p}}{p(p^2+1)}$.

23.8. $\dfrac{p^2+2p+1}{p^3}$.

23.9. $\dfrac{e^{-3p}}{(p+1)(p+3)}$.

23.10. $\dfrac{p^2+5}{(p^2-2p+5)p}$.

23.11. $\dfrac{e^{-2p}(2p-5)}{(p-2)(p-3)}$.

23.12. $\dfrac{e^{-3p}}{p^2-5p+6}$.

23.13. $\dfrac{p^2-2}{(p^2+2p+2)(p^2-2p-2)}$.

23.14. $\dfrac{e^{-p}}{p^2-p-2}$.

23.15. $\dfrac{p^2-4p+6}{(p^2+4p+8)(p+2)}$.

23.23. $\dfrac{e^{-2p}}{p^2-2p+2}$.

23.24. $\dfrac{e^{-p}(p-1)}{p^2-2p-2}$.

23.25. $\dfrac{e^{-3p}(p-2)}{(p-2)^2+4}$.

23.26. $\dfrac{6p^2+2}{(p^2-1)^3}$.

23.27. $\dfrac{p^2+9}{(p^2-9)^2}$.

23.28. $\dfrac{p^3}{(p^2+2p+2)(p^2-2p+2)}$.

23.29. $\dfrac{10p}{(p^2-25)^2}$.

23.30. $\dfrac{p^2+1}{(p^2-2p+2)(p^2+2p+2)}$.

23.31. $\dfrac{2p}{(p^2-2p-2)(p^2-2p+2)}$.

23.32. $\dfrac{p^2+16}{(p^2-2p-2)(p-1)}$.

23.33. $\dfrac{e^{-3p}}{(p-4)(p-3)}$.

23.34. $\dfrac{e^{-p}(p-2)}{p^2+8p+10}$.

23.35. $\dfrac{e^{-p}p}{p^2+p-2}$.

23.16. $\dfrac{e^{-2p}}{(p-1)(p-2)}$.

23.17. $\dfrac{e^{-3p}}{p^2-3p+2}$.

23.18. $\dfrac{p^2+25}{(p^2-25)^2}$.

23.19. $\dfrac{e^{-p}}{p^2-4p+3}$.

23.20. $\dfrac{6p}{(p^2-9)^2}$.

23.36. $\dfrac{e^{-p}(p-3)}{(p-3)^2+4}$.

23.37. $\dfrac{e^{-2p}(p+2)}{p^2-6p+10}$.

23.38. $\dfrac{2p^2+3p+1}{(p^2-2p+5)(p+1)}$.

23.39. $\dfrac{6p+15}{(p-1)(p-3)(p+4)}$.

23.40. $\dfrac{e^{-p}(p+7)}{p^3+1}$.

Задача 24. Операционным методом решить задачу Коши.

24.1. $y''-2y'+y=-12\cos 2x-9\sin 2x,\ \ y(0)=-2,\ y'(0)=0.$

24.2. $y''-6y'+9y=9x^2-39x+65,\ \ y(0)=-1,\ y'(0)=1.$

24.3. $y''+2y'+2y=2x^2+8x+6,\ \ y(0)=1,\ y'(0)=4.$

24.4. $y''-6y'+25y=-24\cos 4x-9\sin 4x,\ \ y(0)=2,\ y'(0)=-2.$

24.5. $y''-14y'+53y=53x^3-42x^2+59x-14,\ \ y(0)=0,\ y'(0)=7.$

24.6. $y''+6y=e^x(\cos 4x-8\sin 4x),\ \ y(0)=0,\ y'(0)=5.$

24.7. $y''-4y'+20y=16xe^{2x},\ \ y(0)=1,\ y'(0)=2.$

24.8. $y''-12y'+36y=32\cos 2x+24\sin 2x,\ \ y(0)=2,\ y'(0)=4.$

24.9. $y''+y=x^3-4x^2+7x-10,\ \ y(0)=2,\ y'(0)=-1.$

24.10. $y''-y=(14-16x)e^{-x},\ \ y(0)=0,\ y'(0)=-1.$

24.11. $y''+8y'+16y=16x^2-16x+66,\ \ y(0)=3,\ y'(0)=0.$

24.12. $y''+10y'+34y=-9e^{-5x},\ \ y(0)=0,\ y'(0)=6.$

24.13. $y''-6y'+25y=(32x-12)\sin x-36x\cos 3x,\ \ y(0)=4,\ y'(0)=0.$

24.14. $y''+25y=e^x(\cos 5x-10\sin 5x),\ \ y(0)=3,\ y'(0)=-4.$

24.15. $y'' + 2y' + 5y = -8e^{-x}\sin 2x,\ \ y(0) = 2,\ y'(0) = 6.$

24.16. $y'' - 10y' + 25y = e^{5x},\ \ y(0) = 1,\ y'(0) = 0.$

24.17. $y'' + y' - 12y = (16x + 22)e^{4x},\ \ y(0) = 3,\ y'(0) = 5.$

24.18. $y'' - 2y' + 5y = 5x^2 + 6x - 12,\ \ y(0) = 0,\ y'(0) = 2.$

24.19. $y'' + 8y' + 16y = 16x^3 + 24x^2 - 10x + 8,\ \ y(0) = 1,\ y'(0) = 3.$

24.20. $y'' - 2y' + 37y = 36e^{x}\cos 6x,\ y(0) = 0,\ y'(0) = 6.$

24.21. $y'' - 8y' = -128x^3 + 48x^2 + 16,\ \ y(0) = -1,\ y'(0) = 14.$

24.22. $y'' + 12y' + 36y = 72x^3 - 18,\ \ y(0) = 1,\ y'(0) = 0.$

24.23. $y'' + 3y' = (40x + 58)e^{2x},\ \ y(0) = 0,\ y'(0) = 2.$

24.24. $y'' - 9y' + 18y = 26\cos x - 8\sin x,\ \ y(0) = 0,\ y'(0) = 2.$

24.25. $y'' + 8y' = -32x^3 + 60x^2 + 18x,\ \ y(0) = 5,\ y'(0) = 2.$

24.26. $y'' - 3y' + 2y = -7\cos x - \sin x,\ \ y(0) = 2,\ y'(0) = 7.$

24.27. $y'' + 2y' = 6x^2 + 2x + 1,\ \ y(0) = 2,\ y'(0) = 2.$

24.28. $y'' + 16y = 32e^{4x},\ \ y(0) = 2,\ y'(0) = 0.$

24.29. $y'' + 5y' + 6y = 52\sin 2x,\ y(0) = 2,\ y'(0) = 6.$

24.30. $y'' - 4y = 8e^{2x},\ \ y(0) = 1,\ y'(0) = -8.$

24.31. $y'' + 16y = (34x + 13)e^{-x},\ \ y(0) = -1,\ y'(0) = 5.$

24.32. $y'' - 3y' + 2y = 3\cos x + 19\sin x,\ \ y(0) = 1,\ y'(0) = 0.$

24.33. $y'' + 5y' = 29\cos x,\ \ y(0) = -1,\ y'(0) = 0.$

24.34. $y'' - 3y' + 2y = 12e^{2x},\ \ y(0) = 2,\ y'(0) = 1.$

24.35. $y'' - y' = 4\cos x + 5\sin 2x,\ \ y(0) = -2,\ y'(0) = -1.$

24.36. $y'' + 6y' + 7y = 3x^2 - 6x + 4,\ y(0) = 3,\ y'(0) = 2.$

24.37. $y'' + 4y' = \cos 5x,\ \ y(0) = 3,\ y'(0) = 2.$

24.38. $y'' + y = \sin x + e^x, \quad y(0) = y'(0) = 1.$

24.39. $y'' - 3y' + 2y = e^{2x}\cos x, \quad y(0) = y'(0) = 2.$

24.40. $y'' + 8y = \mathrm{sh}2x, \quad y(0) = y'(0) = -1.$

Задача 25. Операционным методом найти решение дифференциального уравнения, удовлетворяющее условиям $y(0) = y'(0) = 0$.

25.1. $y'' - y = \dfrac{e^x}{e^x + 1}.$

25.2. $y'' + y = \dfrac{1}{\cos 2x}.$

25.3. $y'' - 4y' + 5y = \dfrac{e^{2x}}{\cos x}.$

25.4. $y'' - y = \dfrac{1}{\mathrm{ch}^2 x}.$

25.5. $y'' + 9y = \dfrac{1}{\cos 3x}.$

25.6. $y'' + 2y' + y = \dfrac{e^{-x}}{(x+1)^2}.$

25.7. $y'' + 2y' + 2y = \dfrac{e^{-x}}{\cos x}.$

25.8. $y'' - 2y' + 2y = \dfrac{e^x}{\cos^2 x}.$

25.9. $y'' + 2y' + 2y = \dfrac{e^{-x}}{\mathrm{ctg}x}.$

25.10. $y'' - 2y + 2y' = \dfrac{e^x}{\cos 2x}.$

25.11. $y'' - 2y' + y = \dfrac{e^x}{x^2 + 1}.$

25.21. $y'' + 4y' + 4y = \dfrac{e^{-2x}}{x^4 - 1}.$

25.22. $y'' - 4y' + 4y = \dfrac{e^{2x}}{x^4 - 1}.$

25.23. $y'' + 2y' + y = 3e^{-x}\sqrt{x+1}.$

25.24. $y'' + y = -\mathrm{tg}^2 x.$

25.25. $y'' - y = e^{2x}\cos(e^x).$

25.26. $y'' - y = e^{2x}\sin e^x.$

25.27. $y'' - y = \dfrac{e^x}{e^x + 1}.$

25.28. $y'' + 4y = \mathrm{tg}^2 2x.$

25.29. $y'' - y = \dfrac{\mathrm{sh}x}{\mathrm{ch}^2 x}.$

25.30. $y'' - 9y = \dfrac{1}{\mathrm{ch}3x}.$

25.31. $y'' + 5y' + 2y = \dfrac{e^{-x}}{\cos 2x}.$

25.12. $y'' + y = \mathrm{tg}2x$.

25.13. $y'' + 4y = \mathrm{tg}2x$.

25.14. $y'' + y = \mathrm{tg}x$.

25.15. $y'' - 2y' + y = \dfrac{e^x}{x-1}$.

25.16. $y'' + 2y' + y = \dfrac{e^{-x}}{x+1}$.

25.17. $y'' + y = \dfrac{1}{\cos x}$.

25.18. $y'' + 49y = \dfrac{1}{\cos 7x}$.

25.19. $y'' + 4y = \dfrac{1}{\cos 2x}$.

25.20. $y'' - 4y = \mathrm{th}2x$.

25.32. $y'' + 2y' + y = \dfrac{e^x}{\mathrm{ch}^2 x}$.

25.33. $y'' + y' = \dfrac{1}{(e^x+1)^2}$.

25.34. $y'' + 2y' = \dfrac{1}{\mathrm{ch}^2 2x}$.

25.35. $y'' + y = \dfrac{e^x}{(e^x+1)^2}$.

25.36. $y'' + 2y' + y = \dfrac{e^x}{x^2+1}$.

25.37. $y'' - 4y = \dfrac{1}{\mathrm{ch}^2 2x}$.

25.38. $y'' - 9y = \dfrac{1}{\mathrm{ch}^2 3x}$.

25.39. $y'' + y = \dfrac{1}{e^x+1}$.

25.40. $y'' + y = \dfrac{e^x}{2+e^x}$.

Задача 26. Операционным методом решить систему дифференциальных уравнений.

26.1. $x' = 2x - y,\ y' = 3x - 2y,\ y(0) = y(0) = 1$.

26.2. $x' = x + y,\ y' = -2x - y,\ x(0) = 1,\ y(0) = -1$.

26.3. $x' = x + y + e^{2t},\ x(0) = 1,\ y(0) = 2$.

26.4. $x' = -x - 2y + 2e^{-t},\ y' = 3x + 4y + e^{-t},\ x(0) = y(0) = -1$.

26.5. $x' = 3x - 4y + e^{-t},\ y' = x - 2y + e^{-t},\ x(0) = -1,\ y(0) = 1$.

26.6. $x' = 4x - y + e^t,\ y' = x + 2y + 3e^{-t},\ x(0) = y(0) = 0$.

26.7. $x' = x - 2y + t,\ y' = x - y + 2,\ x(0) = y(0) = 0$.

26.8. $x' = 4x + 5y + 4,\ y' = -4x - 4y + 4t,\ x(0) = 0,\ y(0) = 3.$

26.9. $x' = 3x + y + e^t,\ y' = -4x - 2y + te^t,\ x(0) = y(0) = 0.$

26.10. $x' = x + y + 3t + 6,\ y' = -10x - y + 6t + 3,\ x(0) = y(0) = 0.$

26.11. $x' = -x - y + e^{2t},\ y' = 2x + 2y + 2e^{2t},\ x(0) = y(0) = 1.$

26.12. $x' = 3x + y + e^t,\ y' = -4x - 2y + te^t,\ x(0) = y(0) = 0.$

26.13. $x' = 2x + \frac{1}{2}y,\ y' = -18x - 4y + 18te^{2t},\ x(0) = \frac{1}{3},\ y(0) = 2.$

26.14. $x' = 7x - 2y + 8te^{-t},\ y' = 8x - y,\ x(0) = 0,\ y(0) = \frac{1}{2}.$

26.15. $x' = -5x - 2y + 24e^t,\ y' = -3x - 4y,\ x(0) = 0,\ y(0) = 2.$

26.16. $x' = -2x - y + 6t,\ y' = -4x - 5y,\ x(0) = 2,\ y(0) = 3.$

26.17. $x' = 5x + 3y,\ y' = -3x - y + 9te^{5t},\ x(0) = \frac{1}{3},\ y(0) = 0.$

26.18. $x' = 11x - 2y + 12te^{-t},\ y' = 18x - y,\ x(0) = -\frac{2}{3},\ y(0) = 0.$

26.19. $x' = -5x - y,\ y' = x - 3y - 36e^{2t},\ x(0) = 1,\ y(0) = -6.$

26.20. $x' = 4x - y,\ y' = x + 2y + 2e^{3t},\ x(0) = 1,\ y(0) = 2.$

26.21. $x' = -2x - y + 37\sin t,\ y' = -4x - 5y,\ x(0) = 0,\ y(0) = -1.$

26.22. $x' = 3x + 2y + (1 - 4t)e^{-t},$

$y' = -2x - 2y + 2te^{-t},\ x(0) = y(0) = -1.$

26.23. $x' = 3x - 5y - 2e^t,\ y' = x - y - e^t,\ x(0) = 2,\ y(0) = 1.$

26.24. $x' = -y,\ y' = -x,\ x(0) = 1,\ y(0) = -1.$

26.25. $x' = -x + y + e^t,\ y' = x - y + e^t,\ x(0) = y(0) = 1.$

26.26. $x' = -y,\ y' = 2x + 2y,\ x(0) = y(0) = 1.$

26.27. $x' + y' - y = e^t,\ 2x' + y' + 2y = \cos t,\ x(0) = y(0) = 0.$

26.28. $x' = x - 2y + t,\ y' = 2x + y + t,\ x(0) = 2,\ y(0) = 4.$

26.29. $3x' + 2x + y' = 1,\ x + 4y' + 3y = 0,\ x(0) = y(0) = 0.$

26.30. $x' = -x + 3y,\ y' = x + y + e^t,\ x(0) = 1,\ y(0) = 1.$

26.31. $x' - y' = 2x - 2y + 1 - 2t,\ x' = -1 - 2y,\ x(0) = y(0) = 0.$

26.32. $x' = x - 2y - 2te^t,\ y' = 5x - y - (2t + 6)e^t,\ x(0) = y(0) = 0.$

26.33. $y' = x + 2y,\ x' = x + 3y + 3,\ x(0) = 0,\ y(0) = 1.$

26.34. $x' = -x + 3y + 2,\ y' = x + y + 1,\ x(0) = 0,\ y(0) = 1.$

26.35. $x' = x + 4y + 1,\ y' = 2x + 3y,\ x(0) = 0,\ y(0) = 1.$

26.36. $x' = 3x + 5y + 2,\ y' = 3x + y + 1,\ x(0) = 0, y(0) = 2.$

26.37. $x' = 3x + 2y,\ y' = 2.5x - y,\ x(0) = 0,\ y(0) = 1.$

26.38. $x' = 3y + 2,\ y' = x + 2y,\ x(0) = -1,\ y(0) = 1.$

26.39. $x' = x + 3y,\ y' = x - 7y,\ x(0) = 1,\ y(0) = 0.$

26.40. $x' = 3x + 2y + e^t,\ y' = 3x - 2y - e^t,\ x(0) = 1,\ y(0) = -1.$